19. PREDICTING DISCONTINUITY IN THE DECISION TO ALLOCATE FUNDS TO CREDIT MEMES WITH A FOKKER-PLANCK EQUATION BASED MODEL[1]

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Abstract

The model is one theoretical approach within a broader research program that could verify the nonlinear conjectures made to quantify and predict potential discontinuous behaviour. In this case, the crisis behaviour associated with financial funds reallocation among various credit instruments, described as memes with the sense of Dawkins, is shown to be of discontinuous nature stemming from a logistic penetration into the behaviour niche. A Fokker-Planck equation description results in a stationary solution having a bifurcation like the solution with evolution trajectories on a 'cusp' type catastrophe that may describe discontinuous decision behaviour.

Keywords: nonlinear models, decision, financial crisis
JEL Classification: C3, C61, C62, D7, D87

Social Reality and Collective Behaviour

I was reading an article by Jeffrey Sachs, in Scientific American, where the text below drew my attention:

"There is little doubt that unduly large swings in macroeconomic policies have been a major contributor to our current crisis. During the decade from 1995 to 2005, then Federal Reserve chairman Alan Greenspan overreacted to several shocks to the economy. When financial turbulence hit in 1997 and 1998—the Asian crisis, the Russian rouble collapse and the failure of Long-Term Capital Management—the Fed increased liquidity and accidentally helped to set off the dot-com bubble. The Fed eased further in 1999 in anticipation of the illusory Y2K computer threat. When it subsequently tightened credit in 2000 and the dot-com bubble burst, the Fed quickly turned around and lowered interest rates again. The liquidity expansion was greatly amplified following 9/11, when the Fed cut interest rates sharply (eventually to a low of

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1 percent in June 2003) and thereby helped to set off the housing bubble, which has now collapsed."

It occurred to me that several credit mimes with the sense of Dawkins are penetrating the niche of the credit portfolio evolving in a logistic way i.e., slowly at the beginning, then bursting to finally saturate. The decision to allocate credit to the new penetrating credit mime may be described by a Fokker-Planck equation whose stationary solution is showing bifurcation behaviour. This model may describe the discontinuous decision to abandon a certain type of credit and allocate the money to the rest of the portfolio. The parameters driving this decision are: cost of risk (potentially measured by, e.g., spread or volatility) and benefit (measured by, e.g., interest).

In a collectivity, individual beliefs, desires and decisions triggering actions are correlated into collective ones.

General sets of social facts may have subsets of institutional facts that result from a collective assignment of functions which may be performed not only by virtue of their physical nature but also by virtue of their social acceptance by the collectivity as having a status assigned to them.

As an example, the noise made by a judge knocking at the end of a trial, without a definite status of behaviour assigned to it, would only represent a natural noise. Conversely, taken inside the set of functions associated with the legal institution, the mentioned noise acquires a special meaning in terms of behaviour.

Thus, in order for functions to have action value that ensures the forming of an institutional structure, a number of reactions need to happen with ideas and functions (described below as memes) that spread among individuals in a collectivity, creating deontic powers such as: rights, duties, obligations, permissions, requirements. Not all of these are only institutional but without them, institutions would not exist as stable dynamic structures (Searle, 2005).

By creating institutional reality, human power is increased by its extended capacity for action. But the possibility to fulfil desires within the institutional structures (like getting rich in an economic structure or becoming president in a political structure) is based only on the recognition, acknowledgement and acceptance of the deontic relationships.

The above statements require a basic intellectual cohesion that connects the members of a collectivity. This involves language as well as the media.

Dynamics of Memes

The movement of ideas and principles among the members of a collectivity creates a dynamic where socio-cultural niches are formed as described by Popper (1973). To look deeper into the dynamic we will consider the memes introduced by Dawkins (the 'virus-like sentences', in Douglas Hofstadter's terminology) that describe the basic conceptual framework for such an analysis (Purica, 1988).

There exists a certain inter-correlation of memes in a society that reacts and diffuses among the individuals. If looking at the financial world one may see it meets the typical criteria described above for a niche where the various financial instruments coexist as financial memes and have a specific dynamics. In what follows we will analyze the way financial memes penetrate and occupy the niche. We must underline that the fact that a given meme species is having a larger part of the niche (call it financial market) depends of the perception by the decision makers of the potential advantage of the instrument. Moreover, it is important to stress that the decision to extend the use of a given instrument is based on perception that feeds from the creation of a collective effect.

The penetration of new financial instruments is done slowly at the beginning and then accelerated, to saturate toward the end. This is typically described by a logistic function. One may notice that this type of behaviour has been shown to describe technology penetration into niches associated to technologies. Marchetti, Nakichenovich, as well as other researchers from IIASA in Laxenburg have extensively done research on the logistic penetration of technologies with important results on the process understanding.

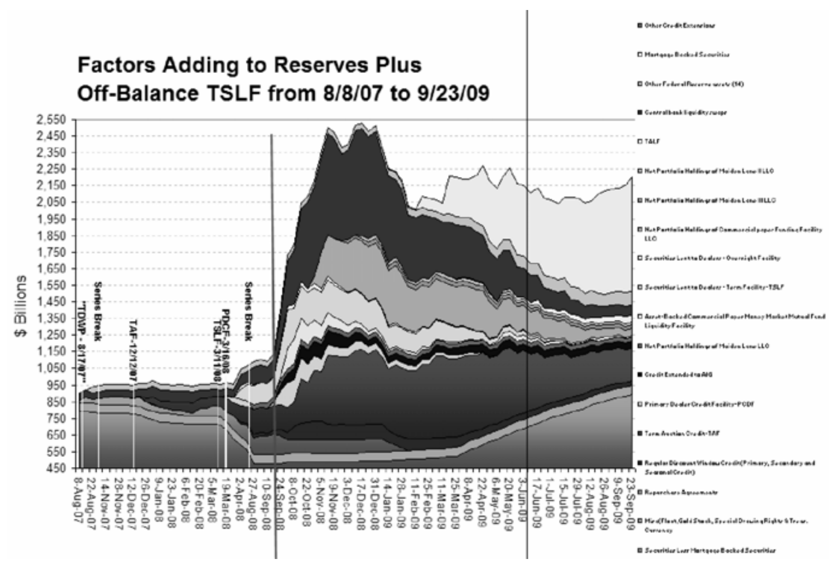
We may note that Marchetti is making an attempt to analyze the evolution of credit taking from banks that identifies some features of the same behaviour, but without identifying the general cause of such behaviour which is related to decision makers' behaviour in the situation of financial institutions. This is where we are bringing some added value in introducing the memes as defined above.

The logistic behaviour is perceived in the case of financial instruments as moments of crisis when a given instrument after having occupied a large portion of the market (niche) saturates, i.e. falls fast.

The data from the 1998 crisis and from the 2008 one are showing the same type of behaviour obviously in relation to different instruments. As we have shown in the beginning of our analysis it seems there are periods of repeated penetration and decrease of some financial meme.

Figure 1

Evolution of various financial instruments in 2007-2008



Source: IAES Conference, Boston, Oct. 2009.

A typical logistic behaviour is seen in the niche of financial instruments used, described as memes, showing increase in the sum allocated to some instruments as they occupy larger parts of the financial market to diminish after saturation and switching to other financial memes.

But in today's world, planning the development of financial systems means having to take into account a large and intricate pattern of various indicators not only connected with economic aspects, but also with the political, sociological, environmental, etc.

The good fitting of the statistical data by the logistic function is only providing the financial planner with a method to predict the evolution of financial instruments penetration in the future. It does not show how to change and control such an evolution. So, it represents just an experimental assertion, a very important and necessary one, but not a consistent theory which would provide the criteria and the means for deciding and influencing the evolution of financial systems.

We are trying to provide a means to fill this gap by analysing the case of fund allocation, measured in monetary units, for the intensification of one or the other of two financial instruments in competition, with the imposed or wished variations of the external parameters represented by indicators of benefits and costs of risk control. This way, development decisions may be taken to avoid sudden, unprepared and large discontinuities resulting in impacts that may affect the evolution of the financial programmes.

Description of the Model

We shall now review the main ideas which have led to the construction of the model; for a more detailed presentation the reader is referred to Gheorghe and Purica (1979).

Experimental grounds

In a 1970 paper, Fisher and Pry tried to fit the statistical data with two-parameter logistic functions of the type:

$$F/(1-F) = \exp(\alpha t + \beta) \tag{1}$$

where: t is the independent variable measuring the time;

α and β are the coefficients of the logistic curves;

F is the portion of the technological market occupied by the new technologies; and

(1- F) is the portion occupied by the old instruments.

This paper was the basis for Marchetti and Peterka, who extended the logistic penetration model to more than two technologies. The same function may be applied to financial instruments. A thorough experimental analysis should be done here as a basis for a research program.

The basic supposition that we extend to memes states that if the substitution of a new instrument for an old one takes place, then the process continues following a logistic curve.

Extending this to a decision level we analyse the decision to intensify one or the other of two financial instruments in competition by transferring monetary funds between their development programmes. The possibility to predict the result of abandoning the funding of one of the programmes is thus opened up, among others, to the decision maker.

The competition between financial instruments in each moment of time is described by a distribution function of the probability to have, at that moment, a certain partition function of the total fund between the two instruments' development programmes.

The distribution function is the stationary solution of a Fokker–Planck equation whose form depends on the expression of the transition probabilities associated with the decision of transferring funds between the two financial instruments.

If we consider that the replacement of an instrument by another is the result of a fund transfer decision from one instrument to the other one, we notice that F/(1-F) represents the frequency of occurrence of the event of partial replacement of an instrument for the other. At the limit we may associate with the decision a transition probability expressed as an exponential function. So, our model is in close resemblance to the Ising model in physics, also used for the description of the polarization of opinion processes in sociology.

Within this approach, the coefficients α, β used in the expression of the transition probabilities have precise meanings, their variation (given or imposed) determining the system's behaviour.

The master equation of the process

We start by admitting that the structure of an economy's financial system may be described as a collection of financial instruments weighted by a given distribution.

In terms relevant to decision-making, the weight of a given instruments or group of financial instruments may be measured by the amount of money put at each moment *t* into the development and deployment of that particular instrument.

For the sake of simplicity we assume that:

- the total amount of money M allocated to the financial sector is practically constant within the observation time lapse;
- we consider only two types of financial instruments.

So, we may write:

$$M=M_1+M_2 \tag{2}$$

with M_i , i=1,2 being the allocated funds for the two types of instruments.

Intensifying, for example, the type one instrument is represented by the transition $(M_1,M_2)\rightarrow(M_1+1,M_2-1)$.

The decision process leading to an allocation is stochastic and cooperative since a given decision is the result of a complex interaction of memes among individuals and/or groups, sensitive to their respective monetary power of influence of each of them, in terms of sound arguments, number, effectiveness, etc., and to external circumstances like market, policy, etc.

That is why one is interested in describing the probability distribution function f to have, at a moment t, decided on an allocation (M_1, M_2).

In order to find it we write the master equation of this process:

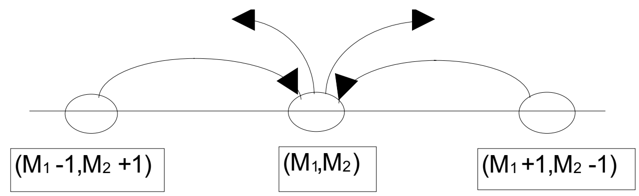
$$f(M_1,M_2,t)/t= w_{21}(M_1+1,M_2-1).f(M_1+1,M_2-1,t)+$$
$$w_{12}(M_1-1,M_2+1).f(M_1-1,M_2+1,t)- \tag{3}$$

$$(w_{21}(M_1,M_2)+w_{12}(M_1,M_2)).f(M_1,M_2,t)$$

which represents the balance between entering the state (M_1,M_2) from and leaving it for the neighbouring states (M_1-1,M_2+1) and (M_1+1,M_2-1); see Figure 2.

Figure 2

Funds allocation



Source: Done by the author.

w_{21} and w_{12} are dependent on the probabilities per unit time of the transitions 1→2 and 2→1, respectively. They are obviously proportional to the effective amount of money in the initial state and must also account for the cooperative character of the decision process underlining transitions.

As we have seen above, the transition probabilities are of exponential form. By similarity to the Ising model in physics we may say that the chance of a 1→2 transition is enhanced by options already made in favour of the 2nd kind of instruments while discouraged by options already made in favour of the 1st kind of instruments. The opposite goes for a 2→1 transition.

With these assumptions w_{12} and w_{21} read:

$$w_{12}(M_1,M_2)=M_1/M \exp[-(1/ \theta).(I.(M_1/M-M_2/M)/2+B)] \quad (4)$$
$$w_{21}(M_1,M_2)=M_2/M \exp[+(1/ \theta).(I.(M_1/M-M_2/M)/2+B)]$$

Identification of the system variable and control parameters

The parameter I is a measure of the intensity of the financial system, B is a ‘preference parameter’ accounting for external influences on the decision process, and θ is a ‘decision climate’ parameter: the higher θ is, the higher the temperature of the debate, owing to a higher perception of the crisis.

Choosing the specific indicators for I and B, in the case of financial instruments, must take into account both the integrative character of these indicators and the scale of magnitude of the financial instruments domain under investigation.

For example, when describing the interplay of two categories of financial instruments one should choose the indicators from the set of financial indicators related to a national financial system.

In order to solve the master equation (3) we define for large M the continuous variable:

$$\xi=(M_1/M-M_2/M)/2 \quad (5)$$

From (5) and (2) we have:

$$M_1/M=1/2+ \xi \quad (6)$$

$$M_2/M=1/2-\xi$$

Expanding all functions of ξ up to the second order in 1/M, one gets a Fokker–Planck-type equation:

$$\delta f/\delta t=-\delta J/\delta \xi \quad (7)$$

where the current J is given by

$$J(\xi)=1/M.(w_{21}-w_{12})f-1/(2M)^2\delta/\delta\xi((w_{21}+w_{12})f) \quad (8)$$

The stationary solution (J=0) is found to be

$$f(\xi)=const.[1/M.(w_{21}(\xi)+w_{12}(\xi))]-1\exp[2M \int\xi^{-1/2}dx[(w_{21}(x)-_{12}(x))/((w_{21}(x)-w_{12}(x))] \quad (9)$$

where:

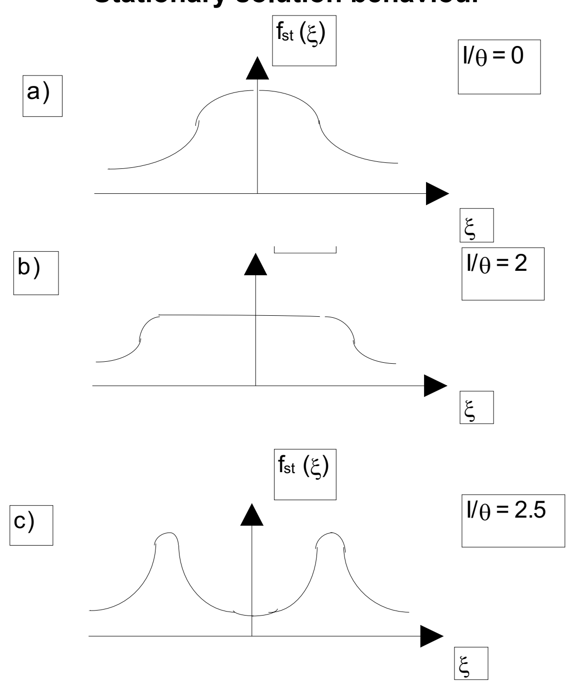
$$w12(\xi)=(1/2+\xi)\exp(-1/\theta.(I.\xi+B))$$
$$w21(\xi)=(1/2+\xi)\exp(1/\theta.(I.\xi+B))$$

Change of stationary solution with variations of the control parameters – the 'cusp' type catastrophe

A numerical analysis of F given by Haken (1975) for B=0 and various I and θ shows some remarkable features (see Figure 3).

Figure 3

Stationary solution behaviour



Source: Done by the author.

1) It says essentially that the higher the financial intensity of a given economy, the higher the chances that strong stands can be taken for more than one financial option.

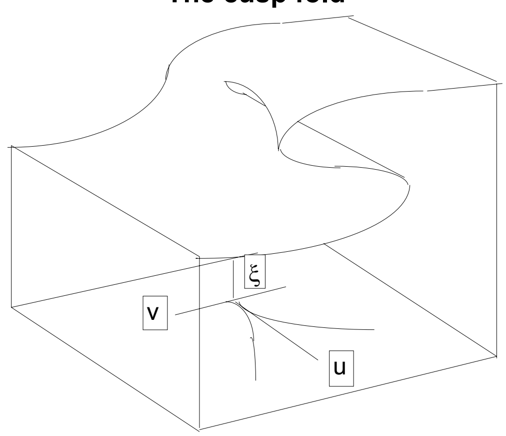
2) And, subsequently, clear decisions are to be expected (sharp probability peaks) when the decision climate is favourable (θ is small, i.e., the 'temperature of the debates' is kept low).

Letting u=I/ θ and v=B/ θ. the function f becomes f(ξ,u,v).

Watching the bifurcation in Figure 3, we may, along with Thom (1972), admit that this shape of the function f implies the existence in the space (ξ,u,v) of a topological surface of the points corresponding to the extreme states of the function f. This surface corresponds to the 'cusp' type catastrophe. Because its points represent the extremes of the function f, which are the equilibrium states of the system, the evolution of the system may be represented by trajectories on this topological surface (see Figure 4).

Figure 4

The cusp fold



Source: Done by the author based on R.Thom.

As it was shown by Thom, this topological surface is determined by a potential whose derivative has the form

$$\delta V(\xi,u,v)/d\,\xi = \xi^{3}+u.\,\xi+v \qquad (10)$$

The projection of the topological surface in Figure 4 on the plane (u,v) is a characteristic bifurcation whose branches represent the limit when a small variation of the parameters implies a sudden change in the system's state, i.e., a sudden large reallocation of funds between the two financial instruments.

The evolution of the financial development programme is represented by the trajectories on the manifold P of the topological surface and their projections on the parameter plane.

Criteria for Financial Development Strategies

Beyond resilience – decisions for safety

Going through the cases described above, we have seen that each sudden jump from one fold to the other means a large reallocation of funds between the two financial instruments. Every such reallocation represents an effort which must be made by that specific institution (even that nation) changing the evolution strategy of its financial systems.

The question arises of what are the amplitude and frequencies of such shocks, which the nation's economy can still absorb and sustain without being completely perturbed.

We have here the very definition of resilience as given by Haefele (1977). But our model goes beyond that by first being able to discern between the amplitudes of shocks and their frequencies of occurrence, thus giving a limit on amplitude – transferred funds – a limit on frequency, and a limit on the total number of shocks. When any of these are reached, the economy is drastically perturbed.

By extending the mechanical analogy we may define a fatigue limit, measured by the number of cyclic shocks (funds reallocation) an economy can sustain before becoming completely exhausted and being forced to change its whole development in order to recover.

Combined amplitude and frequency effects of shocks may be accommodated within our model.

Another feature of this approach is the possibility to predict the arrival of shocks and, based on their predicted amplitude, to decide the most appropriate variations of the control parameters in order to avoid the shock or to mitigate its consequences if it is accepted.

Being able to make such decisions gives the financial system's planner the possibility to optimize the social effort for financial development, thus contributing to an increase in the nation's economic, social and ecological safety.

A Financial Planner's Perception of Risks and Benefits

One important consideration in decisions concerning proposed new financial instruments is whether the risks are out of proportion to those from existing financial instruments which society presently accepts, while 'when seeking to increase the safety (reduce the risk) of existing financial instruments an important factor is deciding where to invest available resources to achieve the greatest reduction in risk'.

Of course in real processes there is a strong interdependence among the total funds invested to enhance financial instruments, the benefits of that instruments and the costs of controlling its risks.

Within this complex dynamical pattern of fast variables, like the funds reallocated at each moment between financial instruments, and slow parameters, such as costs of risk control and benefits, moments may arrive when the planner questions not only where to invest but also when to invest, in order to achieve greater benefits and/or lower costs of risk control.

Situations may occur when the enhancement of instruments temporarily implies greater costs of risk control than those finally reached when the process of instruments development attains a stability stage between the risks control involved for certain benefit. Let us see how these situations can be accommodated by our model.

Dynamic interdependence – development trajectories

First we note that we can no longer consider the control parameters u,v as independent.

Finding a certain dynamic dependence between the parameters also implies the fast variable ξ imposes a flow of trajectories on the smooth surface of the cusp.

We said above that u is a parameter for the 'intensity of the financial instruments' and in what follows we take it as measured by the benefits of those instruments, while v is a 'preference parameter' which we measure by the costs of risk control. We have thus established the control parameters.

The dynamic equations resulting from the interdependence of ξ, u and v are given below for a qualitative example based on the interplay between ξ, u, and v.

We first consider that the time variation (ξ ') of the fast variable is proportional to $\delta V/\delta\xi$; V being the potential corresponding to the cusp catastrophe. In a meme interpretation we believe more in something in which we supported with money and previous trust.

Then we take the variation of the benefits (u') as proportional to the value of the benefit and to the value of the funds allocated to the instruments.

The variation of the costs of risk control (v') depends on the benefits obtained, from which we subtract a constant value v_0, defined as the limit value of the costs of risk control that makes the instruments acceptable to society.

The dynamic system we obtain reads:

$$\begin{aligned} \xi' &= -(\xi^3+\xi.u+v) \\ u' &= a.u+b\,\xi \\ v' &= c.u-v0 \end{aligned} \qquad (11)$$

Figure 5 shows the flow of trajectories on the topological surface.

We see that the equilibrium is given, putting x'=u'=v'=0, and is the point of the coordinates:

$$\begin{aligned} \xi_e &= (av_0)/(bc) \\ u_e &= v_0/c \\ v_e &= -(av_0^2)/(bc^2).((a/b)^2(v_0/c)-1) \end{aligned}$$

The branches of the cusp are given by

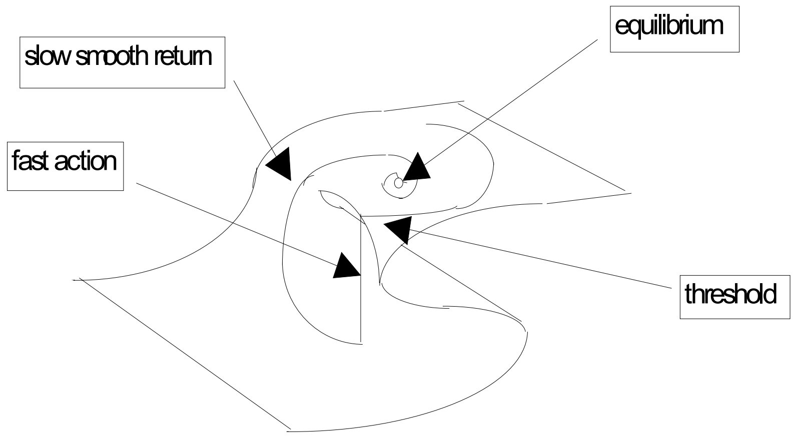
$$4.u^3+27.v^2=0 \qquad (12)$$

In Figure 5 we see that by raising v (costs of risk control), we reach a critical value when a large reallocation of funds to a different type of instruments is necessary.

The critical border is not crossed if the benefits of the first instruments are increased by, for example, finding new applications for it that are of interest to the society.

Figure 5

A flow of trajectories on the cusp fold – potential crisis behaviour



Source: Done by the author.

If we consider a, b, and c constants, we see that the equilibrium point (ξ_e,u_e,v_e) depends on v_0 which is the measure of a society's acceptance of an instrument from the point of view of its cost of risk control.

The example above assumes a linear dependence on u and v.

Even in this rather simple case, it results in a strong dependence of the planner's decision on the society's perception of level of risk (v_0).

It is important to mention that in the conditions above, the equilibrium point is a stable one that all evolution trajectories converge towards. The critical points give an indication of the path to equilibrium, which may be either smooth or discontinuous if the trajectory is crossing from one fold to another.

Perception of Alternatives and Strategic Conduct

To summarize: through a variety of paths, financial decision makers will probably strive toward a status of maximum financial security, or even 'over security' taking advantage of their increased margins of resilience as their financial activity increases.

All paths would, in principle, require timely adjustments in the financial infrastructure of the economy. The amplitude of the adjustment varies with the financial condition of an economy and with the kind of path chosen. If the need for adjustment is ignored, some paths, when followed persistently and blindly, can drive the energy system and the economy into critical states of disruption.

Faced with these findings, it is clear that a prompt and correct perception of the need and the size of structural adjustments in the financial system is a prerequisite for a sound and smooth financial policy.

In terms of this approach, long-term planning of the financial systems means, among other things:

(i) perceiving where one's path is to enter the critical area of the cusp, since, at that moment, the development of potential alternatives to the established financial infrastructure becomes both possible and increasingly advisable;

(ii) perceiving where the respective resilience limit is in order to avoid crossing it through current swings in policy, which may prove costly and painful for society.

Such a generic strategy, advocating R&D programmes, in principle, with not much of an apparent short-term market justification, might have a number of unappealing features. While ignoring the market reality and indulging in wishful thinking is certainly risky, a non-anticipative, non-creative attitude towards the market looks equally unadvisable, for there is also a risk with being too late in giving the green light to latent, sound alternatives that take some unforgiving lead time to develop.

On the other hand, it is only obvious that no need for crash programmes exists when the financial evolution path is chosen appropriately, thus avoiding any accidental and unprepared crossing of the resilience borders of the financial system.

While there is a cost and risk to any change, the fact that there is also a cost and risk to ignoring the need for changes seems inescapable. An attitude that treats alternatives not as subversions of the existing establishment, but as safeguards of its stability and also generators of future, ever more appropriate patterns, is of great importance for both the planner and the public.

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