4. Modeling and Forecasting the Dynamics in Romanian Stock Market Indices Using Threshold Models[1]

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Abstract

We investigate the existence of nonlinear patterns in the dynamics of the main stock index returns in Romania. We use daily closing data of the BET stock index series from 2004 to early 2010. Based on several tests for nonlinearity we reject the null hypothesis of linearity. We use several types of threshold models and compare their fitness and forecasting performance with basic AR models. We found that the LSTAR and SETAR models fit best the data; however, they cannot outperform the simpler AR models in forecasting. These results suggest that although there are nonlinear features in data, the threshold models are not complex enough to reveal the data complexity.

Keywords: Nonlinear Models, Forecasting Models, Threshold Autoregression, Smooth Transition Autoregression, Simulation Techniques

JEL Classification: *C22, C52*

1. Introduction

Many empirical studies have detected significant nonlinearities in the stock prices Hsieh (1991), Abhyankar, Copeland and Wong (1995), Ryden, Teräsvirta and Asbrink (1998). Scheinkman and LeBaron (1989) proved that the daily yields have a strong correlation between consecutive days, but they did not find enough evidence for the presence of chaos in the stock yields. Evidence indicates that the yields are not

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independent and identically distributed (i.i.d.), the cause being not a regime switching or a chaotic dynamic but rather the conditional heteroscketasticity (Hsieh, 1991).

Economic theory suggests a potential number of sources for the presence of nonlinearities in the stock prices: the diversity of the market participants' views (Brock and Hommes, 1998), heterogeneity of the investors objectives given the different views on risk propensity or multiple investment horizons or maybe due to the irrational behavior of the investors and of the mimetic behavior of the investors who follow the general market trend (Lux, 1995).

The financial econometrics of the time series has initially modeled the stock prices as random walk processes implying that that the price changes were i.i.d. Because the assumption was too restrictive, it did not allow the researchers to analyze the financial time series. By relaxing this assumption, it allowed the introduction of nonlinear models in the study of financial time series.

Financial time series have certain peculiar characteristics such as: non-normal distribution of the return (yield), fat tails phenomenon due to the high kurtosis, short-term correlation of return defined as log difference of the daily prices $R_t = \ln(P_t) - \ln(P_{t-1})$, long-term autocorrelation of the volatility, mean-reversion in the securities prices implying a positive autocorrelation on short term and negative autocorrelation on long term (Poterba and Summers, 1988).

Our paper tries to detect if the significant stock market index of Bucharest Stock Exchange (BVB), such as BET stock index as well as its return, manifest nonlinearities described in the above literature review, by applying a series of nonlinearity tests. Since the nature of financial data suggests that nonlinear models are more suitable for describing returns of the financial instruments, we will test several nonlinear models with discrete and smooth transitions in order to detect switching regime behavior of returns.

The paper is organized as follows. We detail the data used in the second section. In the third section we test for the existence of nonlinear features in both log and log-return of data. The estimation of proposed threshold models is done in the fourth section. We compare the quality of the fit and the forecasting capability relative to benchmark AR models. We conclude in the fifth section where we also derive some possible explanations for the results as well as future developments.

2. Methodology

We will proceed in two steps. In the first step we test for the existence of nonlinearities using two of the most common tests for detecting nonlinearities, i.e. Keenan and Tsay tests. In the second step, depending on whether there are or not nonlinearities, we will use nonlinear models, based on the simple AR model, to estimate and forecast the behavior of BET index returns.

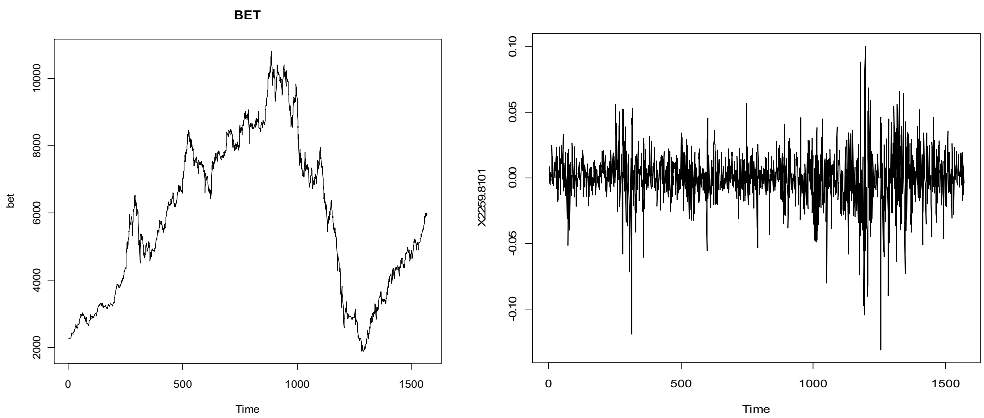
The financial time series are publicly available at the Bucharest Stock Exchange (B.V.B.), website www.bvb.ro. For our analysis we have selected the BET index, which is the reference index for the Bucharest Stock Exchange (BVB) market. The BET index is a price index weighted with the free float of the most liquid 10 companies listed on the regulated BVB market and represents the benchmark for the Romanian capital market.

The data selected comprise daily closing prices of the BET index from January 2004 until March 2010. The reason for selecting 2004 as the starting time was that the Romanian capital market had experienced in 2004 a sharp increase in returns for all BVB indexes, for example the annual return of BET index was 101%, the fixed income instruments underwent a spectacular increase, and the reduction in the trading fees and the investment opportunities attracted many physical persons, which constituted almost 50% of the investors, to trade on BVB. Important events such as the status of functional market economy given by European Commission and the issuance of Law no. 297/2004 on capital market, consolidated the capital market and further attracted foreign investors.

The four main types of non-iid behavior are: 1) linear dependence; 2) nonstationarity; 3) chaos; 4) nonlinear stochastic processes. Because we cannot argue that the behavior of the stock index values remain unchanged for the time interval of approximately six years, we will apply unit root test for detecting nonstationarity of the financial time series analyzed. The unit root test ADF and Phillips-Perron indicate the existence of a unit root for the BET stock indexes. The BET series is made stationary by using the first difference, that is the returns of BET stock index. Also, the Jacques-Bera test of BET index daily closing prices and its returns indicates that the series don't follow a normal distribution.

Figure 1

Daily close and returns for BET and BET-C indexes



Source: Own computations and BVB.

Since the empirical studies on financial time series indicate that the returns of the stock exchange indexes may be nonlinear, we apply Keenan and Tsay tests for detecting nonlinearity. The graphical representation of the BET returns from Figure 1 strengthens our conviction of nonlinearity of the returns, given the occurrence of the classic phenomenon of return clustering around mean which may be attributed to diverse economic factors (Lux, 1995). Also the figure suggests that large returns occur more often than expected, the kurtosis of the returns being fatter than the tails of the normal distribution and that large stock market returns are often negative (due to negative skewness)

3. Testing for the Existence of Nonlinearities

Testing for the existence of nonlinearities is an essential step in the nonlinear modeling of a time series. If there is no evidence of nonlinear dynamics, the nonlinear approach is hardly justifiable and rather classical linear methods should be used. There are several approaches in the literature, but we present and apply two of the most widely used tests, namely Keenan test and Tsay test. The presentation follows the perspective from Cryer and Chan (2008).

Keenan test (1985) is based on a second-order Volterra type expansion. The Volterra expansion, which is similar in concept to the Taylor expansion, is used for nonlinear modeling and its specific feature is its ability to capture memory effects. The Keenan test can be written as:

$$y_t = \mu + \sum_{i=-\infty}^{\infty} \theta_i \varepsilon_{t-i} + \sum_{i,j=-\infty}^{\infty} \theta_{ij} \varepsilon_{t-i} \varepsilon_{t-j} + \sum_{i,j,k=-\infty}^{\infty} \theta_{ijk} \varepsilon_{t-i} \varepsilon_{t-j} \varepsilon_{t-k} + ...,$$

Here $\{\varepsilon_t, -\infty < t < \infty\}$ are a sequence of i.i.d. random variables with mean zero while $y_1,\dots,y_n$ are the observations. The process $\{Y_t\}$ is linear if the double sum of the right-hand side of the equation disappears. Thus, testing the nonlinearity of a series y_t consists practically in testing whether the double sum is zero or not.

Alternatively, as Cryer and Chan (2008) pointed out, Keenan test can also be heuristically derived as follows:

$$Y_t = \theta_0 + \phi_1 Y_{t-1} + ... \phi_m Y_{t-m} + \exp\left\{ \eta \left(\sum_{j=1}^{m} \phi_j Y_{t-j} \right)^2 \right\} + \varepsilon_t$$

where $\{e_t\}$ are independent and normally distributed with zero mean and finite variance. If the regression coefficient $\eta = 0$ then the exponential term becomes 1 and it can be absorbed in the intercept so that the previous model becomes an autoregressive model AR (m). If the regression coefficient η is different from zero, then the previous model is nonlinear. Using the expansion $\exp(x) \approx 1 + x$, which holds for x of small magnitude, we can see that for small η, Y_t follows approximately a quadratic AR model:

$$Y_t = \theta_0 + 1 + \phi_1 Y_{t-1} + ... \phi_m Y_{t-m} + \eta \left\{ \left(\sum_{j=1}^{m} \phi_j Y_{t-j} \right)^2 \right\} + \varepsilon_t$$

The literature has underlined the potential limits of the Keenan test: although it is powerful in detecting nonlinearity in the form of the square of the approximating linear conditional mean function, the strength of the test may be sometimes low, as Keenan has showed (Keenan, 1985).

The test statistic $\hat{F} = \frac{\eta^2(n-2m-2)}{RSS-\eta^2}$ is approximately distributed as an F-distribution with degrees of freedom 1 and n-2m-2.

An enhancement of the Keenan test was provided by Tsay (1986). Again, we follow the description from Cryer and Chan (2008). The Tsay test augments Keenan's approach by replacing the term

$$\eta\left\{\left(\sum_{j=1}^{m}\phi_j Y_{t-j}\right)^2\right\}$$

by

$$\begin{aligned}&\exp(\varsigma_{1,1}Y_{t-1}^2+\varsigma_{1,2}Y_{t-1}Y_{t-2}+\ldots\varsigma_{1,m}Y_{t-1}Y_{t-m}\\&+\varsigma_{2,2}Y_{t-2}^2+\varsigma_{2,3}Y_{t-2}Y_{t-3}+\ldots\varsigma_{2,m}Y_{t-2}Y_{t-m}+\ldots\\&+\varsigma_{m-1,m-1}Y_{t-m+1}^2+\varsigma_{m-1,m}Y_{t-m+1}Y_{t-m}+\varsigma_{m,m}Y_{t-m}^2+\varepsilon_t\end{aligned}$$

Using the approximation we can observe that the nonlinear model is approximately a quadratic AR model but the coefficients of the quadratic terms are unconstrained. The Tsay test considers the following quadratic regression model:

$$\begin{aligned}&Y_t=\theta_0+\phi_1 Y_{t-1}+\ldots\phi_m Y_{t-m}\\&+\varsigma_{1,2}Y_{t-1}^2+\varsigma_{1,2}Y_{t-1}Y_{t-2}+\ldots\varsigma_{1,m}Y_{t-1}Y_{t-m}\\&+\varsigma_{2,2}Y_{t-2}^2+\varsigma_{2,3}Y_{t-2}Y_{t-3}+\ldots\varsigma_{2,m}Y_{t-2}Y_{t-m}+\ldots\\&+\varsigma_{m-1,m-1}Y_{t-m+1}^2+\varsigma_{m-1,m}Y_{t-m+1}Y_{t-m+1}+\varsigma_{m,m}Y_{t-m}^2+\varepsilon_t\end{aligned}$$

and tests whether all m(m+1)/2 coefficients $\varsigma_{i,j}=0$.

To perform tests for nonlinearity we have to specify m, the autoregressive order. Under the null hypothesis that the process is linear, the order can be specified by using the information criterion, for example AIC.

Table 1

Keenan test for the daily time series dlog(BET) (01.2004-03.2010)

Number of lags	F-test	p-value
1	9.09	0.0026
2	13.10	0.0003
3	6.13	0.0133
4	6.29	0.0122
5	6.76	0.0093

Source: Own computations.

Below, we apply the Keenan and Tsay tests to the financial series BET presented in the first section.

Table 2

Tsay test for the daily time series dlog(BET) (01.2004-03.2010)

Number of lags	F-test	p-value
1	7.86	0.0050
2	11.64	1.523e-07
3	9.27	5.255e-10
4	5.81	1.224e-08
5	5.27	2.037e-10

Source: Own computations.

The results for the BET series are consistent and strong in the favor of nonlinearities in the time series over the chosen sample. Both Keenan and Tsay tests for lags 1 to 5 strongly reject the null hypothesis of linearity (we also found the same results even for a larger number of lags). These results allow us to continue our investigation on nonlinear features of the time series from Romanian financial market by using several classes of nonlinear models.

4. Modeling the BET series using Threshold AR Models

4.1. Some theory

The additive nonlinear autoregressive model (AAR, hereafter)

Intuitively, the additive model is just a generalization of the linear regression model. As Hastie and Tibshirani (1990) point out, the additive models retain one important feature of linear models, they are additive in the predictor effects. At the same time, as they underline, the elements must not be either univariate or smooth, as the component functions can have even more than two dimensions.

An additive model, Hastie and Tibshirani (1990), is defined by

$$x_{t+s} = \alpha + \sum_{j=1}^{p} f_j(x_j) + \varepsilon ,$$

where errors are independent of the X_js, $E(\varepsilon) = 0, Var(\varepsilon) = \sigma^2$ and f_js are univariate functions, one for each predictor.

The **Generalized Additive Models** (GAM), see Hastie and Tibshirani (1986), extend the GLM (Generalized Linear Model) approach by replacing the linear form with an additive form. Using scatterplot smoothers allow us to model the terms nonparametrically and let the data suggest the nonlinearities.

The **Additive Nonlinear Autoregressive Model** (AAR) is a *nonparametric additive autoregressive* model (GAM) of the form

$$x_{t+s} = \mu + \sum_{i=1}^{m} s_i (x_t - (i-1)d)$$

where *s* are steps and s_i are nonparametric univariate functions of lagged time series values represented by penalized cubic regression splines. The time delay is *d* and *m* is the embedding dimension, while μ is a constant.

Self-Exciting Threshold AR models (SETAR, hereafter)

Different time series models have been proposed for describing the different regimes generated by a stochastic process. Tong (1978) and Tong and Lim (1980) proposed a threshold autoregressive model (TAR) in which the regime was determined by the value of an observable variable relative to a threshold value.

Our application is based upon first-order SETAR model; see Cryer and Chan (2008) for a detailed presentation. The model is described below:

$$y_t = \begin{cases} \mu_{1,0} + \rho_{1,1}\ y_{t-1} + \sigma_1 e_t \ \text{ if } y_{t-1} < \theta \\ \mu_{2,0} + \rho_{2,1}\ y_{t-1} + \sigma_2 e_t \ \text{ if } \theta < y_{t-1} \end{cases}$$

where: ρ are the autoregressive parameters, σ are noise standard deviations, θ is the threshold parameter and $\{e_t\}$ is a sequence of iid random variables with zero mean and unit variance.

Therefore, if the lag 1 value of y_t is not greater than the threshold, then the conditional distribution of y_t is similar to the first AR(1) process and we say that we are in the lower regime, but when the lag 1 value of y_t is greater than the threshold, then the second AR(1) model is operational and we are in the upper regime. Thus the process switches between two linear models depending on the position of the lag 1 value. Since the error variance may be different in the regimes, the SETAR model can account for conditional heteroscedasticity in the data.

Smooth Threshold AR models (STAR, hereafter)

The transition between one regime to another may not be discrete, as it is in the SETAR model, but smooth. In this respect the switching regression models may be generalized in order to take into account a smoother transition, Terasvirta (1998).

Regarding the financial time series it is more realistic to suppose a continuum of states between the two regimes, Terasvirta (1998). He also pointed to the fact the investors don’t take the same decisions at the same moments, but they rather respond with delays to significant news. The delay may be attributed to interpretation of the news, rebalancing the portfolio and also to their different investment strategies and horizons.

The border between the two regimes in the SETAR model is given by a specific value of the threshold variable y_{t-1}. If the indicator function $I_a\ y_{t-1} > \theta$ is replaced by a continuous monotonic function $G(y_{t-1}, \gamma, \theta)$ of y_{t-1} which changes monotonically from 0 to 1 as y_{t-1} increases, see Franses and van Dijk (2000) or Terasvirta (1994). Then, the resulting model is known as Smooth Transition AR model (STAR). The STAR models have only one function $G(y_{t-1},\gamma,\theta)$ of y_{t-1} that changes.

Logistic STAR Models (LSTAR, hereafter)

If the continuous function G is a logistic function, then the resulting model is called a Logistic Smooth Transition AR model (LSTAR). Our presentation follows Franses and Van Dijk (2000). The transition logistic function is:

$$G(y_{t-1},\gamma,\theta)=\frac{1}{1+\exp(-\gamma[y_{t-1}-\theta])}$$

where location parameter θ determines where the transition occurs and it is interpreted as the threshold between the two regimes corresponding to $G(y_{t-1},\gamma,\theta)$ =0 and $G(y_{t-1},\gamma,\theta)$ =1 as the logistic function changes from 0 to 1. The parameter γ is the slope indicator and it determines the smoothness of the change in the value of the logistic function, namely the speed of the transition from one regime to another. If γ becomes larger, then the transition from 0 to 1 occurs almost immediately at y_{t-1} = θ and, consequently, the logistic function $G(y_{t-1},\gamma,\theta)$ approaches the indicator function I_a $y_{t-1}>\theta$, and the SETAR model may be approximated by a LSTAR model. A two-regime SETAR model is a special case of the LSTAR model when the slope parameter $\gamma\to\infty$ (Terasvirta, 2006).

4.2. Estimation results

In order to assess the performance of the used nonlinear models described above, we use simple linear AR models as a benchmark.

4.2.1. Modeling BET series using SETAR and LSTAR models

SETAR models for BET series

The choice of the number of regimes for SETAR model was set to 2 regimes, after carefully investigating the data. The maximum autoregressive order for the low regime was set at 2, while the maximum autoregressive order for the higher regime was also set at 2. We searched for 1084 possible threshold values within regimes with a sufficient (15%) number of observations and then we searched on 4336 combinations of thresholds (1084), with a threshold delay 1, for a maximum autoregressive order for the low regime 2, and a maximum autoregressive order for the high regime also set at 2. The number of regimes for SETAR model may be explained by fact that the index was not traded as such and there were no derivatives on the BET index at that moment. If the data allow, then the number of regimes changes increases and it is possible to observe more regimes.

The results of the grid search for 1 threshold are presented below in Table 3 after performing a grid search.

Table 3

Grid search for SETAR model (BET series)

Nr.crt	Threshold delay	Low regime	High regime	Threshold	pooled-AIC
1.	0	2	2	0.01333893	8082.412
2.	0	2	2	0.01335794	8081.891
3.	0	2	2	0.01328153	8081.456
4.	0	2	2	0.01318780	8080.338
5.	0	1	1	0.01333893	8079.846
6.	0	2	2	0.01314196	8079.185
7.	0	1	2	0.01333893	8079.011
8.	0	1	1	0.01335794	8078.972
9.	0	1	1	0.01328153	8078.884
10.	0	2	2	0.01355924	8078.508

Source: Own computations.

The best SETAR model (2 regimes) for BET series is described below in the following table.

Table 4

The best SETAR model (BET series)

Coefficients		
Low regime		
phiL.1	phiL.2	Const L
0.036406307	0.116461567	-0.000842535
High regime		
phiH.1	phiH.2	Const H
0.029496587	-0.185539531	0.004717268

Note: Where phiL.1 and phiL.2 are the lower order coefficients for lower regime, constL is the constant for the lower regime. phiH.1 and phiH.2 are the autoregressive coefficients for the higher regime with constH the constant.
Source: Own computations.

No delay was found in the threshold function, following the grid search. The threshold function is given below:

- Variable: Z(t) = + (1) X(t)+ (0)X(t-1)
- Value: 0.0092

We found the threshold value to be positive, and it was estimated at 0.0092, at roughly 1% return. This implies that agents' behavior changes any time the index moves below or above 1% return. The fact that the no delay was found may be explained through the investors' ability to quickly react to the changing market conditions.

The estimated lower regime corresponds to the increasing phase of returns and the upper regime corresponds to the decreasing phase of the returns. When the returns are low, investors are interested to enter the market in order to obtain profit. We see that both coefficients in the lower regime are positive, thus being associated with a robust growth in returns. There is also a significant number of observations in the lower regime (71.71%), while the remaining number in the upper regime (28.29%) is lower than the number in the lower regime, probably speculative or long-term investors. As more and more investors are entering the lower regime driven by the opportunities available on the capital market, the market is cleared and left with fewer attractive investment opportunities. As we enter the upper regime the investors are more vulnerable to higher risks and close their positions. The coefficients in the higher regime indicate decreasing returns for investors. The switching regime indicates that the investors are motivated by quick returns and unwilling to bear higher risks.

LSTAR models for BET series

After performing a grid search for starting values with the following starting values fixed, respectively gamma = 40, threshold = 0.0214; SSE = 0.5554, and the optimization algorithm converged, then the optimized values fixed for regime 2 are gamma = 40 and threshold is 0.02435706. The results are presented in Table 5.

Table 5

Grid search for LSTAR model (BET series)

Nr.crt	Threshold delay	Low regime	High regime	pooled-AIC
1.	0	2	2	-12443.50
2.	0	1	2	-12412.13
3.	0	1	1	-12400.20
4.	0	2	1	-12398.33
5.	1	2	1	-12396.00
6.	1	2	2	-12394.14
7.	1	1	1	-12390.70
8.	1	1	2	-12389.62
9.	2	1	1	-12377.11
10.	2	2	2	-12375.93

Source: Own computations.

Following the results presented in Table 5, the best LSTAR model, according to AIC criterion is the model in position 1. The model is described in Table 6.

Table 6

The best LSTAR model (BET series)

Coefficients		
Low regime		
phi 1.0	phi 1.1	phi 1.2
-0.02821553	-0.42131971	0.26514173
High regime		
phi 2.0	phi 2.1	phi 2.2
0.1019793	-0.4699942	-0.7897399

Source: Own computations.

The estimation of the smoothing parameter was found at a high value, namely that gamma is equal to 40. Such a large value for gamma implies a quick speed of transition between the two regimes.

Again, as in the SETAR case, no delay was found in the threshold function. The threshold function is presented below.

Variable: Z(t) = + (1) X(t) + (0) X(t-1)

Value: 0.02436

In this case, the threshold value was also estimated within the positive range of values, however at a slighter higher value, namely at a 2.4% return.

The estimation results for the SETAR and LSTAR models reveal that investors act quickly to seize investment opportunities as no delay was found in both models and the threshold values are very close.

4.2.2. *Comparing* nonlinear and linear models

After selecting the best SETAR and LSTAR, respectively, within their class of models, we compare these estimated models with benchmark AR models, and a generalized AR model, AAR. We use two basic AR models: the benchmark AR(1) model as well as an AR(2) model. We also used an AAR model as a second class of benchmark models. The tables below compare the results based on two criteria, the AIC criterion, as well as the MAPE criterion for forecasting, both criteria being derived from the model behavior on the sample.

Table 7

Model Comparison for the BET series

	AIC	MAPE
AR(1)	-12377.5	1.475
AR(2)	-12375.8	1.491
SETAR	-12413.2	1.817
LSTAR	-14443.5	1.958
AAR	-12389.2	1.891

Source: Own computations.

Interestingly, the behavior for the time series is quite similar. For both cases, in terms of AIC criterion, the best models are SETAR and LSTAR, with a slight advantage for the LSTAR type model. From the forecasting capability, the ranking changes with the basic AR models ranked as the best. At the same time, the SETAR class has a better forecasting performance relative to the LSTAR or ARR class models.

One possible explanation for this behavior is that the nonlinear models proposed do catch some nonlinearity that exists in the data; however, they are not either nonlinear enough or sufficiently complex in order to catch the changes in the dynamics of series, as seen from the fact that simple AR models outperform them. It is also important to notice that the results in the financial literature reached the conclusion that the baseline AR model remains a powerful benchmark, and a rough expression of financial market movements in the dynamics of returns due to the high correlation in returns between successive days.

5. Conclusion

The recent financial crisis showed that there is always a risk that the market as well as the overall economy start to move unexpectedly and with no connection the previous middle to long run trend. These dynamics ask for more refined approaches than the standard econometric approach. One such framework is threshold autoregressive modeling which allows for a different behavior along each of the regimes assumed.

We investigate the existence of nonlinear patterns in the dynamics of financial data in Romania. We use daily data of the BET series from 2004 to early 2010. Based on several nonlinearity tests, the Keenan and Tsay tests, we reject the null hypothesis of linearity. We use several types of threshold models and compare their fitness and forecasting performance with basic AR models. We found that the LSTAR and SETAR models fit best the data; however they cannot outperform the simpler AR models in terms of forecasting. These results suggest that although there are nonlinear features in data, the threshold models are not complex enough to reveal the data complexity.

The best estimated SETAR and LSTAR models can also be given an economic interpretation. We may think that the estimated lower regime corresponds to the increasing phase of returns and the upper regime corresponds to the decreasing phase of the returns. When the returns are low, investors are interested to enter the market in order to obtain profit. The estimation results for the SETAR and LSTAR models reveal that investors act quickly to seize investment opportunities as no delay was found in both models and the threshold values are very close.

Some future studies should research for models that can reveal better the nonlinearities in the Romanian financial markets.

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