# 4. ASYMMETRIC CONDITIONAL VOLATILITY MODELS: EMPIRICAL ESTIMATION AND COMPARISON OF FORECASTING ACCURACY

**Dumitru MIRON**[*]
**Cristiana TUDOR**[**]

## Abstract[1]

*This paper compares several statistical models for daily stock return volatility in terms of sample fit and out-of-sample forecast ability. The focus is on U.S. and Romanian daily stock return data corresponding to the 2002-2010 time interval. We investigate the presence of leverage effects in empirical time series and estimate different asymmetric GARCH-family models (EGACH, PGARCH and TGARCH) specifying successively a Normal, Student's t and GED error distribution. We find that GARCH family models with normal errors are not capable to capture fully the leptokurtosis in empirical time series, while GED and Student's t errors provide a better description for the conditional volatility. In addition, we outline some stylized facts about volatility that are not captured by conventional ARCH or GARCH models, but are considered by the asymmetric models and document their presence in empirical time series. Finally, we report that volatility estimates given by the EGARCH model exhibit generally lower forecast errors and are therefore more accurate than the estimates given by the other asymmetric GARCH models.*

**Keywords**: stylized facts, leverage effects, asymmetric GARCH, volatility modeling, volatility forecasting

**JEL Classification**: C32, C53

[*] *International Business and Economics Department, Academy of Economic Studies from Bucharest, Romania; e-mail: dumitru.miron@ase.ro*
[**] *International Business and Economics Department, Academy of Economic Studies from Bucharest, Romania; e-mail: cristianat@gmail.com*
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## 1. Introduction

The subject of volatility has always been one of great interest for participants in financial markets, for researchers and even for the general public, often being associated by the latter with the notion of risk. In the current context of an ongoing global financial crisis, terms like volatility forecasting or risk management are nowadays the most important topics in the financial world. There is no doubt that financial market volatility has historically played a crucial role in financial decision making and that volatility forecasts have important applications in areas such as option pricing, hedging strategies, portfolio allocation, as well as Value-at-Risk (VaR) forecasts and optimal capital charges under the Basel Accord.

In this paper we shall discuss some of the most important methods for modeling and explaining volatility behavior, as well as the major procedures for volatility forecasting and forecast accuracy, always with an application on empirical time series from both the US and Romanian markets.

One very important aspect that must always be considered is the fact that financial time series, such as stock returns or exchange rates exhibit some patterns that have been well documented in the literature and that are crucial for correct model specification, as well as estimation and forecasting. This is why we start by presenting below these so-called stylized facts about volatility, before explaining and applying different conditional volatility models on empirical time series.

### *1.1. Stylized facts about volatility in empirical time series*

*Fat tails*

When compared to the normal distribution, the empirical distribution of financial time series exhibits a fourth moment (kurtosis) higher than the normal value of 3 and therefore has fatter tails[2].

*Volatility clustering*

Another stylized fact is the so-called volatility clustering, which refers to the observation of large movements being followed by large movements and is an indication of persistence in past shocks.

*Leverage effects*

First suggested by Black (1976), this styled fact refers to the idea that price movements are negatively correlated with volatility.

*Long memory*

This characteristic means that volatility is highly persistent and there is evidence of near unit root behavior of the conditional variance process. There are two alternative methods of modeling this propriety: a unit root or a long memory process. Both ARCH family and Stochastic Volatility (SV) models use the second approach.

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[2] *In the financial literature the notions of “fat tails” and “heavy tails” are used interchangeably, both referring to leptokurtic distributions. Because they refer to the density of the distribution in the tail area, the term ”heavy” is mathematically more appropriate, although the term “fat” is better suited from a visual point of view.*

*Co-movements in volatility*

When analyzing time series from different markets, one observes that big movements in one financial time series is matched by big movements in another time series from a different market.

In order to get reliable forecasts of future volatility, researchers must consider and incorporate in their models these stylized facts.

### *1.2. Literature review*

Two of the most widely used models in investigating return volatility are the Autoregressive Conditional Heteroskedasticity (ARCH) and Generalized ARCH (GARCH) developed by Engle (1982), and extended by Bollerslev (1986) and Nelson (1991). Two important characteristics found when investigating financial time series, heavy tails and volatility clustering, i.e. the tendency for large (small) swings in prices to be followed by large (small) swings of random direction, can be captured by the GARCH family models. It is important to point out that, unlike other models for volatility, ARCH models formulate conditional variance of returns via maximum likelihood procedure, instead of using sample standard deviations.

Many empirical studies in the financial literature found support for the ARCH/GARCH models and their extensions. Akgiray (1989) finds GARCH consistently outperforms other models in all sub-periods and under all evaluation measures while Pagan and Schwert (1990) find EGARCH is best especially in contrast to nonparametric. Cao and Tsay (1992) document that E-GARCH gives the best forecast for small stocks which they explain by a leverage effect. Bali (2000) documents the usefulness of GARCH models, the nonlinear ones in particular, in forecasting one-week-ahead volatility of U.S. T-Bill yields.

In general, models that allow for volatility asymmetry came out well in the forecasting contest because of the strong negative relationship between volatility and shocks. Charles Cao and Ruey Tsay (1992), Ronald Heynen and Harry Kat (1994), Lee (1991), and Adrian Pagan and G. William Schwert (1990) favor the EGARCH model for volatility of stock indices and exchange rates. On the other hand, Brailsford and Faff (1996) and Taylor (2001) find GJR-GARCH to outperform GARCH in stock indices.

Franses and Van Dijk (1998) study the performance of the GARCH model and two of its non-linear modifications, respectively the Quadratic GARCH of Engle and Ng (1993) and the Glosten, Jagannathan and Runkle (1992) model to forecast weekly stock market volatility. They find that the QGARCH model is best when the estimation sample does not contain extreme observations such as the 1987 stock market crash and that the GJR model cannot be recommended for forecasting.

Hansen and Lunde (2005) compare 330 ARCH-type models in terms of their ability to describe the conditional variance. The models are compared out-of-sample using DM-$ exchange rate data and IBM return data, where the latter is based on a new data set of realized variance. The authors find no evidence that a GARCH(1,1) is outperformed by more sophisticated models in the analysis of exchange rates, whereas the GARCH(1,1) is clearly inferior to models that can accommodate a leverage effect in the analysis of IBM returns.

Donaldson and Kamstra (1997) document important differences between volatility in international markets, such as the substantial persistence of volatility effects in Japan relative to North American and European markets.

De Santis and Imrohoroglu (1997) investigate stock returns and volatility in emerging financial markets and find clustering, predictability and persistence in conditional volatility, as others have documented for mature markets. However, they document that emerging markets exhibit higher conditional volatility and conditional probability of large price changes than mature markets.

Nam, Pyun and Arize (2002) use asymmetric nonlinear smooth-transition (ANST) GARCH(M) models and find that, for monthly excess returns of US market indexes over the period 1926–1997, negative returns on average reverted more quickly, with a greater reverting magnitude, to positive returns than positive returns revert to negative returns.

If the volatility of the US stock market, as well as other developed and emerging markets has long been investigated, there are not many studies on the Romanian financial market.

Harrison and Paton (2004) use data on stock markets in two transition economies (Romania and the Czech Republic) to demonstrate the importance of using the correct GARCH specification. They show that, when returns are characterized by ‘fat tails’ or kurtosis the use of a GARCH-t specification is appropriate.

Lupu (2007) calibrates an EGARCH (Exponential GARCH) model for the logarithmic returns of the Romanian composite index BET-C. The article provides the testing of the predictive power of the model by estimating the model and then evaluating its performance on an out of sample test.

Tudor (2008) employs GARCH-family models to investigate the Risk-Return Tradeoff on the Romanian stock market. Various time series methods are employed, including the simple GARCH model, the GARCH-in-Mean model and the exponential GARCH. Results of the study confirm that E-GARCH is the best fitting model for the Bucharest Stock Exchange composite index volatility in terms of sample-fit.

Syllignakis and Kouretas (2008) use weekly stock market data to examine whether the volatility of stock returns of ten emerging capital markets of the new EU member countries (including Romania) has changed as a result of their accession in the EU. They find that the high volatility of stock returns of all new EU emerging stock markets is associated mainly with the 1997-1998 Asian and Russian financial crisis while there is a transition to the low volatility regime as they approach the accession to EU.

Finally, Tudor (2008) employs symmetric GARCH models to investigate the volatility on the Romanian and American stock markets. The paper considers two empiric time series from each market (the composite index BET-C and TLV stock for Bucharest Stock Exchange and the S&P 500 index and the Coca-Cola stock for New York Stock Exchange) and finds that the volatility of the TLV Romanian stock TLV cannot be modeled by GARCH, while the symmetric GARCH models are correctly specified for S&P 500 and KO. For BET-C the estimated models did not remove all heteroskedasticity from the residuals, suggesting that other specifications for the variance equation must be found.

In this paper, we continue the preliminary analysis from Tudor (2008) by investigating the presence of asymmetric effects in empirical time series of stock returns from the Romanian and US stock markets. We begin with a brief review of the asymmetric GARCH-family statistical models. Further, we investigate the presence of leverage effects in empirical time series. Afterwards, for those financial series where evidence for asymmetry is found, asymmetric GARCH models for conditional volatility (such as E-GARCH, T-GARCH and P-GARCH) are further estimated. Different specifications for the error term distribution are further considered until models that best capture conditional volatility characteristics for each return series are selected. Afterwards, forecasting ability of the four asymmetric GARCH models is investigated. Some concluding remarks are given in the end of the paper.

## 2. Asymmetric GARCH-class models: empirical estimation

### *2.1. An overview on the asymmetric conditional volatility models*

The simple GARCH model, besides its main virtue which consists in its simplicity, has also two important shortcomings. On one hand, it can be hard to fit, especially when more than one lag on each variable is involved. On the other, it also restricts the impact of a shock to be independent of its sign, whereas there is evidence of an asymmetric response for some markets, notably the stock market. Stock return volatility increases following a sharp price drop, but a price rise of the same size may even lead to lower volatility.

Indeed, in the basic GARCH model only squared residuals enter the conditional variance equation, therefore the signs of the residuals or shocks have no effect on conditional volatility. However, a stylized fact of financial volatility is that bad news (negative shocks) tends to have a larger impact on volatility than good news (positive shocks). In other words, volatility tends to be higher in a falling market than in a rising market. Based on this conjecture, the asymmetric news impact on volatility is commonly referred to as the leverage effect (Zivot (2008)).

Nelson (1991) proposed a GARCH-class model named Exponential GARCH that allows for asymmetric effects and therefore solves one of the important shortcomings of the symmetric models. While the GARCH model imposes the nonnegative constraints on the parameters[3], EGARCH models the log of the conditional variance so that there are no restrictions on these parameters:

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[3] *The GARCH (p,q) model for the conditional volatility is written as:* $\sigma_t^2 = \omega_0 + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2$*, where* $\alpha_i>0$ *and* $\beta_j>0$ *to allow the conditional variance* $\sigma^2_t$ *to be always positive. Also,* $\sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \beta_j < 1$ *is the condition for stationarity. The conditional variance equation is therefore a function of three terms: the mean* $w_0$*, news about volatility from the previous period, measured as the lag of the squared residual from the mean equation (the ARCH term) and last period's forecast variance (the GARCH term).*

$$\log\left(\sigma_t^2\right) = \omega + \beta \log\left(\sigma_{t-1}^2\right) + \alpha \left|\frac{\varepsilon_{t-1}}{\sigma_{t-1}}\right| + \gamma \frac{\varepsilon_{t-1}}{\sigma_{t-1}}$$

Note that the left-hand side is the logarithm of the conditional variance. This implies that the leverage effect is exponential, rather than quadratic, and that forecasts of the conditional variance are guaranteed to be nonnegative.

The presence of leverage effects can be tested by the hypothesis that $\gamma < 0$. If $\gamma \neq 0$, then the impact is asymmetric.

E-GARCH basically models the log of the variance (or standard deviation) as a function of the lagged logarithm of the variance/std dev and the lagged absolute error from the regression model. It also allows the response to the lagged error to be asymmetric, so that positive regression residuals can have a different effect on variance than an equivalent negative residual.

Another extension of the classic GARCH model that allows for leverage effects is the Threshold-GARCH. The idea of the Threshold ARCH (or TARCH) model is to divide the distribution of the innovations into disjoint intervals and then approximate a piecewise linear function for the conditional standard deviation (Zakoian (1994)), and the conditional variance respectively (Glosten *et al*. (1993)).

Rabemananjara and Zakoian (1993) extend this preliminary Threshold model by including the lagged conditional standard deviations (variance respectively) as a regressor, which is known as the TGARCH model. They also give conditions for covariance-stationarity in their study.

T-GARCH is therefore estimated with the following equation:

$$\sigma_t^2 = a_0 + \sum_{i=1}^{p} a_i \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \gamma_i S_{t-i} \varepsilon_{t-i}^2 + \sum_{j=1}^{q} b_j \sigma_{t-j}^2,$$

where:

$$S_{t-i} = \begin{cases} 1 & \varepsilon_{t-i} < 0 \\ 0 & \text{if } \varepsilon_{t-i} \geq 0 \end{cases}.$$

In other words, depending on the $\varepsilon_{t-i}$ being above or under the *threshold value* (which equals zero), $\varepsilon^2_{t-i}$ will have different effects on the conditional variance $\sigma^2_t$, as it follows:

- When $\varepsilon_{t-i}$ is positive, total effects are given by $a_i\, \varepsilon^2_{t-i}$;
- When $\varepsilon_{t-i}$ is negative, total effects are given by $(a_i + \gamma_i)\varepsilon_{t-i}^2$.

This is why in the case of TGARCH we expect $\gamma_i$ to be positive, so that bad news would have a more powerful effect on volatility than good news.

Another well-known asymmetric GARCH-family model is the PGARCH (*Power GARCH*) developed by Ding, Granger and Engle (1993).

The model they proposed (PGARCH (p,d,q)) has the following equation:

$$\sigma_t^d = a_0 + \sum_{i=1}^{p} a_i (|\,\varepsilon_{t-i}\,| + \gamma_i \varepsilon_{t-i})^d + \sum_{j=1}^{q} b_j \sigma_{t-j}^d,$$

where *d* is a positive coefficient and $\gamma_i$ represents leverage effects.

When d = 2, the above equation becomes a classic GARCH model that allows for leverage effects and when d = 1 the conditional standard deviation will be estimated. In addition, we can increase the flexibility of the PGARCH model by considering *d* as another coefficient that must also be estimated [see Zivot (2008)].

## 2.2. Estimation of asymmetric conditional volatility models on empirical time series

### 2.2.1. Data and methodology

We consider two empirical time series from each market (US and Romania), consisting in stock indices returns, as well as individual stock returns. For each market we chose a comprehensive and diversified stock index along with a well-known individual stock. For Bucharest Stock Exchange, we consider the composite index BET-C and the IMP stock (Impact Bucharest), a company listed on the first tier at BSE. BET-C is a market capitalization weighted index and reflects the price movement of all the companies listed on the BSE regulated market, Ist and IInd Category, excepting the SIFs.

For New York Stock Exchange we consider the Standard and Poor's index and also the KO stock (Coca-Cola). S&P 500 is an equity value-weighted arithmetic index and in mid-1989 represented 76 percent of the equity capitalization of the NYSE. Daily returns are computed as logarithmic price relatives: $Rt = \ln(P_t)/\ln(P_{t-1})$, where *t P* is the daily price at time *t*.

The approach taken in this paper is one-step-ahead forecasts. One-step-ahead prediction is useful in evaluating the adaptability of a forecasting model. Since our main goal is to evaluate the volatility forecasting performance of different asymmetric volatility models, we wish to consider a reasonably large hold-out sample. Therefore, the sample data set is divided into two parts. The first part covers a seven years period (January 02, 2001-February 09, 2008) and comprises daily observations, totaling a number of 1853 observations for each empirical time series, or a total of 7412 daily observations. The second part covers the period February 09, 2008-February 08, 2010), or 433 daily observations for each financial series. We use the first part of the data set for estimating the initial parameters of the models, while the second part of the data set serves for producing out of sample forecasts. To assess the forecasting performance of various models, we need to compare forecasted volatilities with actual volatilities. Unfortunately, the actual volatility is not directly observed and hence it has to be estimated. A common approach in the literature is to use the absolute or squared daily return to estimate the daily volatility. We will follow the latter convention and use squared log-returns as a proxy for volatility in our study.

### 2.2.2. Preliminary investigations and Empirical characteristics

Table 1 presents the summary statistics (mean, standard deviations, skewness, kurtosis, Jarque-Bera normality test and ADF unit root test) for daily stock returns.

We notice that the daily return of the Romanian composite index BET-C had a mean value of 0.124% during the considered period, well above the returns of the other financial assets, while its daily volatility represented by the standard deviation (1.31%)

is above the volatility of the two American series, but smaller than the volatility of IMP stock.

Furthermore, as mentioned before, we present four statistics which are calculated using the observations in the full sample: Skewness, Kurtosis and the Jarque-Bera normality test. The skewness coefficient is negative for three out of the four time series (with the exception of S$P500), suggesting that the three series have a long left tail while kurtosis is very high in all cases (from 5.79 for S$P 500 to an extreme value of 80.82 for IMP a reflection that all distributions are highly leptokurtic. As expected, the Jarque-Bera test rejects normality at the 5% level for all series.

**Table 1**

**Descriptive statistics and preliminary investigations**

| | BET-C | IMP | KO | S&P 500 |
|---|---|---|---|---|
| Mean | 0.001240 | 0.000339 | 6.64E-05 | 1.98E-05 |
| Median | 0.000664 | 0.000000 | 0.000000 | 0.000118 |
| Maximum | 0.062457 | 0.139762 | 0.053273 | 0.055744 |
| Minimum | -0.102876 | -0.622378 | -0.105973 | -0.050468 |
| Std. Dev. | 0.013187 | 0.034257 | 0.011917 | 0.010548 |
| Skewness | -0.464047 | -5.057259 | -0.661180 | 0.056924 |
| Kurtosis | 8.040739 | 80.82222 | 10.07767 | 5.794280 |
| Jarque-Bera | 2028.295* | 475495.3* | 4002.639* | 603.8436* |
| ADF Unit Root Test | -35.47425* | -38.92070* | -45.30357 | -42.31412* |

* Significant at 0.01.

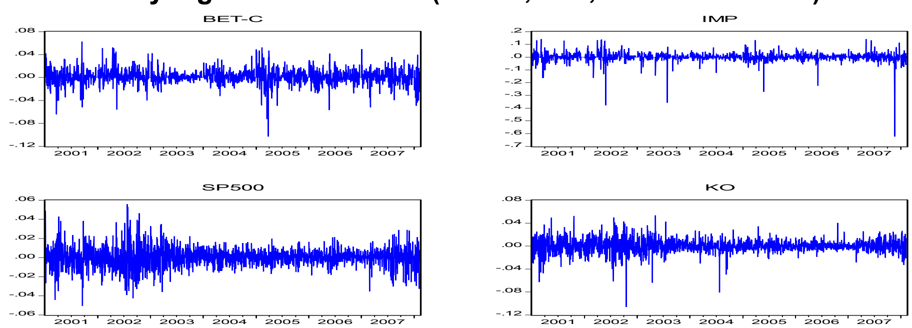
** ADF test is conducted at level and the lag length is chosen automatically, based on Schwartz Information Criterion with the maximum number of lags allowed equal to 24.

Finally, the stationarity of the four series is tested by conducting the Augmented Dickey Fuller Unit Root Test. The specifications of the ADF tests were given by the graphical representation of each of the return series (trend, intercept, no trend and intercept). In all four cases, the null hypothesis of a unit root is rejected. We therefore conclude that the time series are stationary at level and we can proceed to model the conditional volatility with GARCH-class models.

Next, Figure 1 presents the evolution of daily logarithmic returns of the four series during the considered time period. We observe that volatility clustering seems to be present in all cases, with the possible exception of the Romanian stock IMP. A series with some periods of low volatility and some periods of high volatility is said to exhibit volatility clustering. Volatility clustering can be thought of as clustering of the variance of the error term over time: if the regression error has a small variance in one period, its variance tends to be small in the next period, too. In other words, volatility clustering implies that the error exhibits time-varying heteroskedasticity (unconditional standard deviations are not constant). In conclusion, the time-series plot of the daily returns in Figure 1 clearly shows the familiar volatility clustering effect, along with a few occasional very large absolute returns, more so in the case of IMP stock.

**Figure 1**

**Daily logarithmic returns (BET-C, IMP, S&P500 and KO)**




Further, we test for asymmetric effects on conditional volatility in the four financial series investigated. A simple diagnostic for uncovering possible asymmetric effects is the sample correlation between squared returns and lagged returns, or Corr($r^2_t$, $r_{t-1}$) [see Zivot (2008)]. A negative value for this correlation coefficient provides evidence for potential leverage effects. Table 2 presents estimates of this coefficient for the four time series. We notice that the correlation between $r^2$ and $r_{t-1}$ has a small negative value in all cases, indicating weak evidence for asymmetry. Asymmetric GARCH models could therefore perform well in explaining conditional volatility for the four financial series.

**Table 2**

**Testing for asymmetric effects on conditional volatility for the four empirical time series**

| Series | Corr ($r^2_t$, $r_{t-1}$) |
|---|---|
| BET-C | -0.064103 |
| IMP | -0.054781 |
| S&P 500 | -0.076307 |
| COCA-COLA | -0.077161 |

*2.2.3. Model estimates*

First, we filter conditional mean structure in the data by estimating ARMA(p,q) models with AR(p) and MA(q) orders determined by AIC. The results from mean equations (not presented here) show generally significant parameters for lagged returns but insignificant parameters for lagged errors, indicating that the best suited model for the mean equation is an AR(1,) model for all time series. We used ACF, PACF and the Q test to test for any remaining serial correlation in the mean equation and to check the specification of the mean equation. We conclude that the mean equation is correctly specified, as all Q-statistics are not statistically significant.

Furthermore, we estimate a series of asymmetric GARCH-family models to explain conditional variance and volatility clustering for each of the four series: EGARCH (1, 1), TGARCH (1,1) and PGARCH (1,d,1), for d = 1, 2. Parameter estimates are reported in Table 3.

**Table 3**

**Parameter estimates of asymmetric GARCH models**

| Conditional Volatility Model | *C* | *ARCH(-1)* | *GARCH(-1* | $\gamma_1$ |
|---|---|---|---|---|
| **BET-C** | | | | |
| EGARCH | -2.20263 (0.0000) | 0.384592 (0.0000) | 0.781001 (0.0000) | -0.073308 (0.0000) |
| TGARCH | 2.40E-05 (0.0000) | 0.199268 (0.0000) | 0.615096 (0.0000) | 0.138734 (0.0000) |
| PGARCH2 | 2.41E-05 (0.0000) | 0.264638 (0.0000) | 0.614187 (0.0000) | 0.132520 (0.0000) |
| PGARCH1 | 0.002966 (0.0000) | 0.238949 (0.0000) | 0.591016 (0.0000) | 0.182367 (0.0000) |
| **IMP** | | | | |
| EGARCH | -3.667686 (0.0000) | 0.449178 (0.0000) | -0.237577 (0.0000) | 0.505156 (0.0000) |
| TGARCH | 0.000386 (0.0000) | 0.135619 (0.0000) | 0.404536 (0.0000) | 0.485432 (0.0000) |
| PGARCH2 | 0.000385 (0.0000) | 0.334011 (0.0000) | 0.361860 (0.0000) | 0.405473 (0.0000) |
| PGARCH1 | 0.012596 (0.0000) | 0.275923 (0.0000) | 0.461376 (0.0000) | 0.465636 (0.0000) |
| **S&P 500** | | | | |
| EGARCH | -0.20523 (0.0000) | 0.064212 (0.0000) | 0.983489 (0.0000) | -0.112774 (0.0000) |
| TGARCH | 9.96E-07 (0.0000) | -0.021700 (0.0000) | 0.946638 (0.0000) | 0.126625 (0.0000) |
| PGARCH2 | 1.17E-06 (0.0000) | 0.030717 (0.0000) | 0.930263 (0.5527) | 0.911049 (0.5634) |
| PGARCH1 | 0.000162 (0.0000) | 0.052584 (0.0000) | 1.000000 (0.0000) | 0.942371 (0.0000) |
| **COCA-COLA** | | | | |
| EGARCH | -0.15726 (0.0000) | 0.099124 (0.0000) | 0.990578 (0.0000) | -0.058158 (0.0000) |
| TGARCH | 1.27E-06 (0.0000) | 0.011829 (0.0000) | 0.942468 (0.0000) | 0.079203 (0.0000) |
| PGARCH2 | 1.28E-06 (0.0000) | 0.042162 (0.0000) | 0.942442 (0.0000) | 0.468554 (0.0000) |
| PGARCH1 | 9.58E-05 (0.0000) | 0.054729 (0.0000) | 0.951922 (0.0000) | 0.564532 (0.0000) |

*Note*: p-values associated with the Student test are presented in parentheses.

With only one exception (for the PGARCH2 model estimated for S&P 500) the coefficients that reflect leverage effects ($\gamma_1$) are statistically significant at 1% for all series.

The value of the statistically significant coefficient $\gamma$ indicates the magnitude of the leverage effect, and the sign of its direction. If results agree that asymmetric effects are present for the four time series, they are mixed in what the direction of the asymmetry is concerned.

EGARCH models show a negative and significant $\gamma$ parameter for BET-C, Coca-Cola and S&P500 suggesting that past negative shocks have a greater impact on subsequent volatility than positive shocks do. On the contrary, for the Impact stock the leverage coefficient is positive and also statistically significant, showing that future stock volatility is greater influenced by past positive events.

TGARCH leverage effects are positive and significant for the four series, attesting that bad news increase volatility.

The results of the estimation of PGARCH models confirm that the asymmetric effects are present for BET-C, IMP, KO and S&P500. PGARCH1 and PGARCH2 $\gamma$ coefficients are positive and significant (with the one exception mentioned before), though with the opposite sign than expected. The positive innovations would imply a higher next period conditional variance than negative innovations of the same sign, indicating that the existence of leverage effect is not observed in returns of the four stock market return series.

The mixed results concerning the leverage effect encountered in our analysis are not quite unusual as shown by Glosten, Jagannathan and Runkle (1993) and cited by Zlatko (2007). They provided a brief overview of the conflicting results in the literature and then explained why both positive and negative relationship between past returns and subsequent volatility would be consistent with theory.

We can attest that asymmetric effects are indeed present on the US and Romanian return series, thus we expect the asymmetric GARCH family models to perform better than a simple symmetric GARCH in explaining conditional volatility for the considered time series.

In Table 4 we present information criteria and the log-likelihood function for the estimated asymmetric models and also for a simple GARCH (1,1) model estimated for the same financial series. Results confirm that, as expected, asymmetric models have both smaller values for information criteria and bigger log-likelihood functions than the simple GARCH(1,1) (with the exception of the Romanian index BET-C and common stock Impact, for which the symmetric model outperforms two of the asymmetric ones).

Information criteria show that the asymmetric PGARCH2 is the best in explaining conditional volatility for the Romanian index BET-C and also for the Romanian stock IMP, EGARCH is the most suited for S&P 500, and PGARCH1 has the best specifications for explaining Coca-Cola's conditional volatility.

Table 4

**Information criteria and log-likelihood function for symmetric and asymmetric GARCH models**

| *Model* | *AIC* | *BIC* | *Log-likelihood* |
|---|---|---|---|
| **BET-C** | | | |
| GARCH | -6.02717 | -6.01822 | 5587.173 |
| EGARCH | -6.00047 | -5.98854 | 5563.440 |
| TGARCH | -6.03247 | -6.02055 | 5593.091 |
| PGARCH2 | -6.03256 | -6.02063 | 5593.171 |
| PGARCH1 | -6.00447 | -5.99255 | 5567.147 |
| **IMP** | | | |
| GARCH | -4.096805 | -4.087861 | 3798.690 |
| EGARCH | -4.088361 | -4.076436 | 3791.867 |
| TGARCH | -4.109550 | -4.097525 | 3810.591 |
| PGARCH1 | -4.088502 | -4.076576 | 3791.997 |
| PGARCH2 | -4.109569 | -4.097644 | 3811.516 |
| **S&P 500** | | | |
| GARCH | -6.54375 | -6.53481 | 6065.792 |
| EGARCH | -6.58939 | -6.57746 | 6109.071 |
| TGARCH | -6.58844 | -6.57651 | 6108.191 |
| PGARCH1 | -6.58733 | -6.57541 | 6107.168 |
| PGARCH2 | -6.58410 | -6.57217 | 6104.171 |
| **COCA-COLA** | | | |
| GARCH | -6.23872 | -6.22978 | 5783.180 |
| EGARCH | -6.26482 | -6.25290 | 5808.364 |
| TGARCH | -6.25499 | -6.24306 | 5799.251 |
| PGARCH2 | -6.25494 | -6.24301 | 5799.204 |
| PGARCH1 | -6.26573 | -6.25380 | 5809.202 |

Finally, we re-estimate the models after having eliminated the restrictive assumption that the error terms follow a normal distribution. In order to accomplish this goal, we assume that residuals follow successively a Student distribution and also a Generalized Errors Distribution (or GED), two of the distributions capable of incorporating "fat tails" usually present in empirical distributions. Therefore, we estimate both the simple GARCH (1,1) and the best fitted asymmetric GARCH-class model for each of the four time series considering first that the residuals follow a Student distribution, and after that a GED. Table 5 presents AIC, BIC and log-likelihood functions in all cases.

Table 5

**Information criteria and log-likelihood function for estimated non-Gaussian GARCH models**

| Model | AIC | BIC | Log-likelihood | AIC | BIC | Log-likelihood |
|---|---|---|---|---|---|---|
| ***BET-C*** | ***Student Distribution*** | | | ***GED*** | | |
| GARCH | -6.138227 | -6.126301 | 5691.067 | -6.1438 | -6.1318 | 5696.234 |
| PGARCH2 | -6.139696 | -6.124789 | 5693.428 | -6.14595 | -6.1310 | 5699.224 |
| ***IMP*** | ***Student Distribution*** | | | ***GED*** | | |
| GARCH | -4.796309 | -4.78438 | 4447.781 | -4.47007 | -4.4611 | 4144.525 |
| PGARCH2 | -4.798431 | -4.78352 | 4450.746 | -4.46944 | -4.4575 | 4144.943 |
| ***S&P 500*** | ***Student Distribution*** | | | ***GED*** | | |
| GARCH | -6.571648 | -6.55972 | 6092.632 | -6.57918 | -6.5672 | 6099.618 |
| EGARCH | -6.595090 | -6.609997 | 6129.162 | -6.613870 | -6.5989 | 6132.751 |
| ***COCA-COLA*** | ***Student Distribution*** | | | ***GED*** | | |
| GARCH | -6.348831 | -6.33690 | 5886.192 | -6.34300 | -6.3310 | 5880.793 |
| PGARCH1 | -6.36466 | -6.34976 | 5901.866 | -6.35745 | -6.3425 | 5895.183 |

Results in Table 5 confirm that in two out of four cases (for BET-C and S&P 500), the best specifications are found in models that consider GED for the error terms' distribution, while for the two time-series of common stocks (Impact and Coca-Cola) a Student distribution brings better results. Nevertheless, for all series a non-Gaussian distribution of the error terms is more appropriate than the very restrictive normality assumption. One should notice that in all cases the log-likelihood function for the asymmetric GARCH is higher than for the corresponding symmetrical one. Only in the case of IMP (GED) we find similar values for the log-likelihood function, which determined the information criteria to favor the symmetric model.

In conclusion, the best specifications for modeling and explaining conditional volatility of the four stochastic processes are: PGARCH(1,2,1) model with GED standard errors for BET-C, PGARCH(1,2,1) model with Student's t standard errors for Impact, EGARCH(1,1) model with GED standard errors for S&P 500 and PGARCH(1,1,1) model with a Student distribution for residuals in the case of the Coca-Cola stock.

Parameter estimates of the above presented specifications together with the ARCH LM test that investigates any signs of heteroskedasticity left in the residuals are presented in Table 6. For the ARCH LM test, the null hypothesis investigated is that there are no more ARCH effects in the residuals and the value of p (the number of lags) used in running the test is 20. For all time series, the LM test results validate the homoskedasticity assumption (last column in Table 6) and attest that all volatility models are correctly specified.

**Table 6**

**Estimated parameters and ARCH-LM test for the best specified conditional volatility models**

| Time Series | Model | C | ARCH (-1) | GARCH (-1 | $\gamma_1$ | GED/ DOF | Log likelihood | BIC | ARCH LM |
|---|---|---|---|---|---|---|---|---|---|
| **BET-C** | PGARCH (1,2,1) | 1.83E-05 (0.0000) | 0.35208 (0.0000) | 0.607035 (0.0000) | 0.117526 (0.02) | 1.05376 (0.0000) | 5699.2 | -6.13 | 1.02985 (0.4220) |
| **IMP** | PGARCH (1,2,1) | 0.242138 (0.0000) | 0.37211 (0.0000) | 0.36192 (0.0000) | 0.565840 (0.00) | 2.00018 (0.0000) | 4450.704 | -4.78 | 0.270961 (0.9999) |
| **S&P 500** | EGARCH (1,1) | -0.1932 (0.0000) | 0.06809 (0.0002) | 0.985057 (0.0000) | -0.11805 (0.00) | 1.42396 (0.0000) | 6132.7 | -6.59 | 0.847490 (0.6559) |
| **Coca-Cola** | PGARCH (1,1,1) | 8.67E-05 (0.0062) | 0.05374 (0.0000) | 0.952965 (0.0000) | 0.497242 (0.00) | 5.22677 (0.0000) | 5901.8 | -6.34 | 0.556495 (0.9422) |

NOTE: - p-values are presented in parentheses;

## 3. Forecasting Performance of asymmetric GARCH Models

Besides estimation, the other important application and use of conditional volatility models is for forecasting volatility. Accurate volatility forecasts are important to different categories of participants in the financial world, such as traders, investors, risk managers or researchers and estimates of future volatility are critical inputs in both option pricing models and value-at-risk models.

Figlewski (2004) shows that all ARCH-type models share three significant shortcomings as forecasting tools. First, they all seem to need a large number of data points for robust estimation. Second, they are subject to the general problem that the more complex any model is and the larger the number of parameters it involves, the better it will tend to fit a given data sample, and the quicker it will tend to fall apart out-of-sample. Thirdly, these models essentially focus on variance one step ahead and are not designed to produce variance forecasts for a long horizon. When trying to forecast more than a few periods ahead, the forecasts can not incorporate any new information from the (unknown) future disturbances, and will simply converge to the long run variance at a rate that depends on the value of $(a_1 + b_1)$.

If in the previous section we selected the most adequate model for each financial asset in terms of criteria based on the goodness of fit of the candidate models, in this section we will select the best model on the basis of the forecasting performance of the candidate models.

To this end, we consider the squared log-returns as a proxy for the actual volatility and proceed to producing one-day out-of-sample volatility forecasts for each of the four investigated time series. GARCH models are estimated using a moving window of 6 years of data (1853 daily observations), as in the previous section. We start with the sample ranging from the first trading day of January 2001 until the 8$^{th}$ of February 2008. The estimated models then are used to obtain 1- step-ahead forecasts of the conditional variance during the next trading day of February 2008 (11th). Next, the window is moved one day into the future, by deleting the observation from the first trading day of January 2001 and adding the first forecasted observation. As we roll over the sample, the various GARCH models are re-estimated on this sample, and are used to obtain further forecasts. This procedure is repeated until the final estimation sample consists of the last window of 1853 observations ending on February 8th 2010.

To evaluate and compare the forecasts from the different models, several evaluation criteria are computed, with true volatility measured by the squared realized log-return.

In the literature a variety of statistics have been used to evaluate and compare forecast performance. These include root mean square error (RMSE), mean absolute error (MAE), mean absolute percentage error (MAPE), mean mixed error (MME), the Theil-U statistic, and the LINEX loss function. Among these, RMSE, MAE and MAPE are conventionally used error statistics. Similar to Bluhm and Yu (2001), we argue that for practitioners like options traders relative profits and thus relative volatility are often more important than absolute values. Therefore, we use MAPE to measure the performance of out-of-sample model forecast results, defined as follows:

$$MAPE = \frac{1}{T}\sum_{t=1}^{T}\frac{\left|\hat{\sigma}_{t+h} - \sigma_{t+h}\right|}{\sigma_{t+h}},$$

where: T is the number of out-of-sample observations minus the number of days of the forecast horizon; $\sigma_t$ the actual volatility at the period t; $\hat{\sigma}_t$ is the forecasted volatility at the period t.

Table 7 reports in Panel A the mean absolute percentage error (MAPE) for each of the asymmetric GARCH models and each financial time series for the out-of-sample period February 9th 2008 to February 8th 2010.

Similar to Brailsford and Faff (1996), each error statistic is also expressed on a relative basis where the benchmark is the value of the statistic for the worst performing model for each time series (Panel B).

**Table 7**

**Out-of-sample mean absolute percentage errors for asymmetric conditional volatility models**

| **Conditional Volatility Model** | ***BET-C*** | ***IMP*** | ***S&P 500*** | **KO** |
|---|---|---|---|---|
| **PANEL A: Mean Absolute Percentage Error (MAPE) - Actual** | | | | |
| EGARCH | 0.39629 | 0.43598 | 0.44865 | 0.47560 |
| TGARCH | 0.54390 | 0.66732 | 0.46768 | 0.66103 |
| PGARCH2 | 0.47653 | 0.41753 | 0.65498 | 0.78532 |
| PGARCH1 | 0.68965 | 0.63578 | 0.71534 | 0.64289 |
| **PANEL B: Mean Absolute Percentage Error (MAPE) - Relative** | | | | |
| EGARCH | 0.574625 | 0.65333 | 0.627184 | 0.605613 |
| TGARCH | 0.788661 | 1 | 0.653787 | 0.841733 |
| PGARCH2 | 0.690974 | 0.625682 | 0.915621 | 1 |
| PGARCH1 | 1 | 0.952736 | 1 | 0.818634 |

An examination of Table 7 reveals that EGARCH models exhibit generally lower forecast errors. For three out of the four series, respectively BET-C, S&P500 and KO we find that EGARCH is associated with lower actual values for MAPE. Only for the Romanian stock IMP we find that PGARCH2 produces the most accurate out-of-sample forecasts, while EGARCH ranks second. The PGARCH1 models are the worst performing two times, respectively in the case of stock market indices BET-C and S&P500 (see Panel B).

While the MAPE estimates for all four GARCH models may be considered as high in absolute terms, they only intend to give a relative indication of overall forecasting performance. Nevertheless, our reported results are lower than those given by Brailsford and Faff (1996), although higher than results found in Balaban (2000) or Balaban, Bayar and Faff (2006).

In summary, although a different conditional volatility model was found best suited for each series in terms of in-sample modeling, on an out-of-sample basis EGARCH clearly dominates the other models, ranking first in terms of forecasting performance in three cases and second in the fourth case. Thus, the MAPE statistic clearly identifies the EGARCH model as superior. Nevertheless, caution should be used in the interpretation of these results, as a change in sample size, rolling window, forecast horizon, frequency of observations and other variables could greatly impact the above findings.

## 4. Conclusions

Modeling and forecasting volatility in financial markets has always been an important subject of inquiry and research in the literature and its relevance has even increased nowadays in the very turbulent financial world.

In this paper, our goal was to compare various models of stock volatility both in terms of sample fit and out-of-sample forecasting performance. Taking the 2001-2010 period as a sample and using daily observations for four different return series from the Romanian and US stock markets, we conclude that the asymmetric GARCH-family models give a better explanation of returns' volatility than the simple GARCH model. For each of the four time series, we find the corresponding asymmetric conditional volatility model that has the best suited specifications.

In addition, we have examined the empirical performance of the asymmetric GARCH models for forecasting volatility in US and Romanian stock markets and found that mean absolute percentage errors are substantially lower for volatility forecasts conducted with the Exponential GARCH model.

Summing up, the results presented in this paper confirm previous findings in the literature that return series are uncorrelated in time, but they present the phenomenon of volatility clustering. In addition, we can report that GARCH family models with normal errors are not capable to fully capture the leptokurtosis in empirical time series, while GED and Student's t errors provide a better description for the conditional volatility. Finally, asymmetric effects are present in empirical data and asymmetric models that are capable of allowing different responses to different past shocks perform better in explaining conditional volatility.

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