6. AN EVALUATION OF EFFECTIVENESS OF FUZZY LOGIC MODEL IN PREDICTING THE BUSINESS BANKRUPTCY

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Abstract

In front of the current global financial crisis, the future existence of the firms is uncertain. The characteristics and the dynamics of the current world and the interdependences between the financial and economic markets around it demand a continuous research for new methods of bankruptcy prediction. The purpose of this article is to present a fuzzy logic-based system that predicts bankruptcy for one, two and three years before the possible failure of companies. The proposed fuzzy model uses as inputs financial ratios, that is dynamics of the financial ratios. In order to design and to implement the model, authors have used financial statements of 132 stock equity companies (25 bankrupt and 107 nonbankrupt). The paper presents also the testing and validation of the created fuzzy logic models.

Keywords: bankruptcy, crisis, prediction, fuzzy logic, ratings
JEL Classification: G17

I. Introduction

Starting business, no one assumes to bring the company into bankruptcy process, but to reach growth/development. So, why companies bankrupt? According to the literature, it is not possible to indicate one reason that would be 100% responsible for the bankruptcy of a company (Dahiya and Klapper, 2007), (Dyrberg, 2005), (Richardson, Nwankwo, *et al*., 1994). The firms failure is a result of the whole set of factors. These factors very often are overlapping, even with different sources of origin - endogenous and exogenous.

Nowadays, in times of uncertainty, risks and incomplete information, the crisis becomes a feature of modern business, not the state of emergency. Crises are an

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inherent part of every company (Daianu and Lungu, 2008). Financial crisis does not appear suddenly in the company - from day to day, but it is the result of accumulation of many factors and symptoms of the deteriorating economic situation ignored by managers over a longer period of time in the company and its environment. In most cases, bankruptcy is a continuous process, where it is possible to distinguish several stages - from the emergence of the first signs of the financial crisis, through blindness, ignorance of the financial and nonfinancial symptoms of crisis in the firm, wrong activities until the final phase of the crisis, which is bankruptcy (Ooghe and Prijcker, 2006; Moulton and Thomas, 1996). The process of going bankrupt may take even up to 5-6 years. This is not a sudden phenomenon, impossible to predict. Therefore, the earlier warning signals are detected, the more time managers have for preparing and reacting in the next phases of the crisis. In the theory of economy, business bankruptcies are considered in two aspects: macro and micro. From the point of view of the macro aspect, the failures of companies are treated as a positive factor of the free market - which is - elimination of not efficient companies. On the other hand, from the micro point of view, the bankruptcies result in a number of negative economic and social effects for:

- owners of these companies,
- banks and firms that gave the credits to failed companies,
- cooperation companies which would also suffer by losing business partners (consequently, it could jeopardize financial situation of nonbankrupt company),
- employees who lose their jobs.

That is why the topic of predicting the possible failure of the firms in a few years before the bankruptcy occurs is very significant. The purpose of this article is to propose a fuzzy logic-based system, having as inputs financial ratios and dynamics of the financial ratios, to predict bankruptcy one year, two years and three years before the possible failure of the company. Additionaly, the performance of the presented bankruptcy prediction system is evaluated, presenting the results comparatively to those obtained using a fuzzy logic model with only financial ratios as inputs.

II. Characteristics of bankruptcy prediction methods

Much research in predicting the failure of firms has been done using financial statement data. The primary focus has been on using standard statistical techniques in which it appears that the financial ratios are generally valid discriminators between bankrupt and nonbankrupt firms. The extensive study of bankruptcy prediction reveal that the most popular methods used for prediction were discriminant analyses (30.3% of all models), then logit and probit models (21.3%). Therefore, the main method regarding this issue has been discriminant analysis (DA). Less popular methods used in bankruptcy prediction are: logit and probit models, decision trees, artificial neural networks, etc. (Aziz and Dar, 2004).

There are two types of discriminant analysis used for bankruptcy prediction: univariate and multivariate. Univariate DA models evaluate the financial situation of the company based on every financial ratio separately. Multivariate DA models take into account all ratios in one model at the same time, so at the end there is one score that classifies a firm as a bankrupt or nonbankrupt company. The precursor of the usage of multivariate discriminant analysis for business bankruptcy prediction was E. Altman, who developed the following model based on 66 US companies in 1968 (33 bankrupt and 33 nonbankrupt) (Altman, 2006):

$$Z = 1,2 * X_1 + 1,4 * X_2 + 3,3 * X_3 + 0,6 * X_4 + 0,999 * X_5$$

where:
X1 = working capital / total assets
X2 = retained earnings / total assets
X3 = earnings before taxes / total assets
X4 = market value of equity / total long term and short term liabilities
X5 = sales / total assets

E. Altman proposed to use three decision areas depending on the value of the Z score:

- if Z < 1,81 then it is signal of high probability of going bankrupt,
- if 1,81 < Z < 2,99 then the risk of financial failure of the company is not possible to define (it is so called “gray area”),
- if Z > 2,99 then there is low probability of the bankruptcy.

One of the newest discriminant analysis models for forecasting business bankruptcy of Central European companies is the model of T. Korol. The following functions of the multivariate disciminant model were estimated (if Zban>Znon then the company is classified as the enterprise at the bankruptcy risk, if Zban<Znon then such firm is classified as a good one) [Korol, 2011]:

$$Zban = -1.97 + 2.35\ X1 - 2.90\ X2 - 2.68\ X3 + 0.79\ X4$$
$$Znon = -3.49 + 9.93\ X1 - 0.05\ X2 - 0.62\ X3 + 1.19\ X4$$

Where: X1 = Profit from sales / total assets

X2 = working capital / total assets

X3 = (net profit + amortization) / Long term and short term liabilities

X4 = Operating costs / short term liabilities

The discriminant analysis requires certain very restrictive assumptions. The modeler is required to specify the precise relationship between inputs and outputs and any restrictions that may be implied by theory – e.g. normal distribution of the ratios used in the model.

The additional disadvantage of parametric (e.g. discriminant analysis model, logit and probit models) and most nonparametric bankruptcy prediction models is the necessity of using the 50%:50% proportion of bankrupt and nonbankrupt companies in the learning dataset. This raises the assumption that in the testing dataset there is also such a proportion of “good” and “bad” firms. In case of testing such models when most of the companies are not in danger of failure, the models will generate more errors by classifying nonbankrupt companies as a bankrupt ones. If the modeler uses in learning dataset the proportion, e.g. 10% bankrupt and 90% nonbankrupt firms, the created models will not have the ability of recognizing future bankrupt firms. The author of this article, T. Korol, has verified the influence of proportion of bankrupt and nonbankrupt companies in learning and testing dataset on the effectiveness of predictions in his previous research (Korol, 2005). Examples of nonparametric models are based on artificial neural networks (Nastac, Dobrescu, *et al.*, 2007, Morariu, Iancu, *et al.*, 2009, Korol, 2005).

Although artificial neural network models are the most popular among soft computing techniques, they make up only 9 per cent of usage of all models (statistical and soft computing). Among artificial neural network models, the most well-known is the one developed by Wilson and Sharda for evaluating American companies. The parameters of their model are as follows [Wilson, Sharda, 1994]:

- 5 neurons in the entry layer (the same financial ratios that are used by E. Altman $X_1....X_5$),
- 10 neurons in the hidden layer,
- 2 neurons in the output layer (BR – bankrupt = 0, NBR – nonbankrupt = 1),
- network is trained by the use of a backpropagation algorithm.

In case of using the expert system based on the fuzzy logic, there is no necessity of conducting the "learning process" of the model. The model is created by the expert based on his knowledge and experience. This allows to test the programmed model in the situation close to real one, in which there are more nonbankrupt companies than bankrupt in the testing dataset.

Although the concept of fuzzy sets was introduced by Zadeh as early as 1965, the use of fuzzy logic in predicting business bankruptcies was practically unknown until 2006. Since 2007 there were published only a few papers describing the possibility of implementing such fuzzy system in forecasting this negative phenomenon in firms. According to Purvinis, Virbickaite, *et al*. (2008) and Balcaen and Ooghe (2006), the literature does not provide a clear image regarding the applications of alternative methods (including strategies based on expert systems) used for business failure prediction and therefore further research is necessary. According to Purvinis, Virbickaite, *et al*. (2008) and Chena, Huangb, *et al*. (2009), failure prediction methods based on fuzzy logic are more useful to managers than methods based on neural networks that are hardly interpretable (an explanation of a specific forecast cannot be provided using neural networks). Neural networks are useful to refine the knowledge base of the expert system when it is necessary. The available studies in the literature prove that there still are a lot of questions and issues unsolved. Recent studies showed a lot of imperfections in the process of forecasting with the fuzzy logic. Accessible results are not satisfactory. In Purvinis, Virbickaite, *et al*. (2008), a fuzzy modeling attempt of the bankruptcy dependence of Lithuanian enterprises is provided. The idea presented by Purvinis, Virbickaite, *et al*. (2008) is appreciable, but the presentation or the study seems to be incomplete. The study of Purvinis, Virbickaite, *et al*. (2008) focuses on establishing fuzzy rules using different indicators, therefore associating output membership functions. The authors believe that the study should focus more on the parameters of the membership functions associated to the inputs and to try to reduce the number of rules. In Purvinis, Virbickaite, *et al*. (2008), the bankruptcy prediction is realized only one year before possible financial failure and additionally authors only focus mainly on the use of the ratios connected to the Golden Balance Rule and the structure of liabilities (for example, there are no profitability ratios). The prediction results have also to be improved. Authors consider that associating a pair of S-shaped and Z-shaped membership functions is enough for most input indicators, and the membership functions from Chena, Huangb, *et al*.

(2009) and Purvinis, Virbickaite, *et al*. (2008) are raising the complexity of the model without being necessary.

Approaches like Chena, Huangb, *et al*. (2009), Thomaidis, Gounias, *et al*. (1999), Spanos, Dounias, *et al*. (1999) are using fuzzy logic-based prediction systems using as inputs only financial ratios. In Chena, Huangb, *et al*. (2009) and Spanos, Dounias, *et al*. (1999) it is mentioned that the results of the prediction based on fuzzy logic are better than using classical models. The authors consider that the main drawback of these studies is that only financial ratios are considered as inputs and therefore the quality of the bankruptcy prediction is limited. In studies of Thomaidis, Gounias, *et al*. (1999) and Spanos, Dounias, *et al*. (1999), the authors have used a quite broad spectrum of financial ratios, but their results of forecast are less than satisfactory. In Spanos, Dounias, *et al*. (1999) the classification results for holdout sample for all three years prior to bankruptcy were at the level of 63.16%, while the type II error was as big as 63.15% (the type II error refers to the classification of a non-bankrupt firm within the bankrupt class). Such effectiveness is much lower than the results generated by the traditional simple forecasting models as discriminant analysis models or logit models accessible in the literature. Also the studies of Thomaidis, Gounias, *et al*. (1999) are characterized by the low prediction accuracy of bankruptcy. In this case the authors got effectiveness of about 30% in the analysis of three years prior to failure and about 40% accuracy in the analysis of two years before bankruptcy. Therefore, it can be assumed that the analysts are still in the preliminary stage of implementing the fuzzy logic in this field of finance. Hence, there is growing need to research further this method of forecast of bankruptcy of companies.

T. Korol proved in his research the superiority of the multi-criteria system in which the core model was based on fuzzy logic methodology over the statistical models such as the one of discriminant analysis presented in this section (Korol, 2011). Therefore, the aim of this paper is further research on the use of fuzzy logic in forecasting business bankruptcy. Authors investigate the effectiveness of individual fuzzy logic model (without any supporting models as in the mentioned multi-criteria system) and without any additional macroeconomic supporting information as presented in Korol and Korodi (2010). Such research approach will let checking for effectiveness and usefullness of fuzzy logic to predict bankruptcy solely based on the financial ratios of the analyzed companies. This will increase the usability of such bankruptcy prediction model as the analysts can implement it directly in companies in different countries without the need of adaptating it to the macroeconomic conditions of each region of the world. Additionally authors also verify the effectiveness of the dynamic fuzzy logic model (using also as inputs dynamics of the financial ratios) and compare it to the effectiveness of the static fuzzy logic model (using as inputs only financial ratios).

III. Basic fuzzy logic concepts

The majority of the systems functioning nowadays, in all the areas, must have decision capabilities (e.g. control engineering, finance, law, etc.). They have to be able to provide an answer to a considered question. Some of them are using classical (conventional) logic, which will always provide affirmative or non-affirmative answers, meaning “white” or “black”, “no” or “yes”, “small” or “large”, etc. These sets of answers are considered to be the set of two truth values {0, 1}. The basic logical operations that can be applied to these values are the AND, OR and NOT operators. In propositional logic every atomic sentence is evaluated using operators based on the three basic operators mentioned before and a corresponding true or false (0 or 1) value is associated. The complex sentences are formed by atomic sentences and according to the propositional logic, the result of their evaluation can lead only to true or false value assignments.

The characteristic function of a classical set A for all $x \in X$ will be {1} if $x \in A$ and {0} otherwise, respectively.

The classical set theory operations are realized using classical logical operators.

As presented among others in Korol and Korodi (2010), in many cases the {0, 1} types of answers are not presenting enough relevance, or they are not even needed. The reason is that many questions cannot be answered correctly only through two fix values. Vagueness and impreciseness are components of everyday life. In order to characterize certain situations there is a need of proper decisions that are taking into consideration possible middle values (e.g. middle values between "small"/"large", "black"/"white", etc.).

The idea behind the fuzzy logic theory is to replace the set of truth values {0, 1} with the entire interval [0, 1], practically to take a much more complex decision.

A fuzzy set of a universe X is represented by a membership function that maps each element to a degree of membership to the interval [0, 1]. The membership function is a generalized form of the characteristic function and it is associated to the fuzzy logic.

Considering the "small"/"large" example, a sentence in this universe according to the classical logic theory can have two possible values, but using the fuzzy logic theory the provided answer may have a big (maybe infinite) number of values, evaluated in the following manner "how large/small regarding the largest/smallest". The fuzzy sets are therefore solving the problem of vague linguistic terms.

The evaluation in this example realized using the classical logic theory (a characteristic function) and the fuzzy logic theory (a membership function), respectively, is illustrated in Figure 1 and Figure 2.

Figure 1

Classical logic, a characteristic function

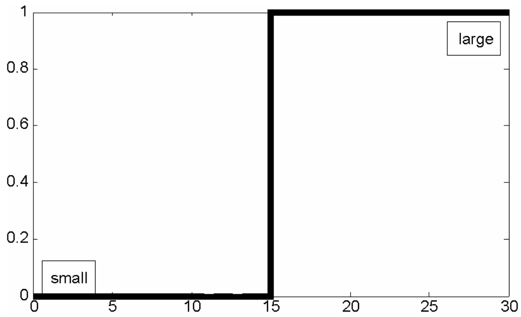
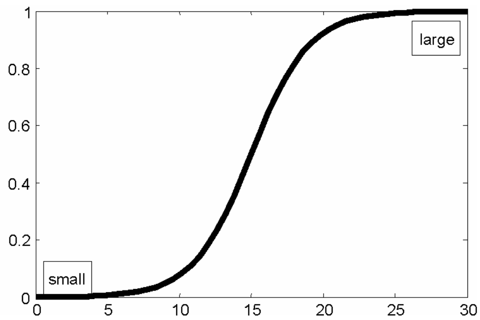


Figure 2

Fuzzy logic, a membership function



The fuzzy logic is based on *IF condition THEN conclusion* type rules. These rules are involving vague linguistic expressions modeled by fuzzy sets.

Considering the general set of rules of the following example, as being the core of a fuzzy decision system,

IF ($condition_{11}$) AND/OR ($condition_{21}$) … ($condition_{n1}$) THEN $conclusion_1$

IF ($condition_{12}$) AND/OR ($condition_{22}$) … ($condition_{m2}$) THEN $conclusion_2$

…

IF ($condition_{1p}$) AND/OR ($condition_{2p}$) … ($condition_{lp}$) THEN $conclusion_p$,

where each of the conditions involves a considered input of the system, the entire fuzzy logic procedure is divided into certain steps:

a) to provide the input values to the fuzzy system,
b) to evaluate each condition and provide the corresponding truth values,
c) to provide the conclusion for each rule,
d) to provide the general conclusion for the complete set of rules,
e) to provide an output value of the fuzzy system.

In the first two steps, a) and b), after obtaining the fuzzy sets by associating the corresponding membership functions, there is a need to evaluate the conditions within the rules. In order to evaluate the conditions, generalized basic logical operators are applied: conjunction (AND), disjunction (OR), negation (NOT). The standard operators which are satisfying the standard requirements for fuzzy conjunction, disjunction and negation are the triangular norm (t-norm), triangular conorm (t-conorm) and negation.

The well-known four standard t-norms and t-conorms are:

$$T_{\mathbf{M}}(x,y) = \min(x,y) \qquad S_{\mathbf{M}}(x,y) = \max(x,y)$$

$$T_{\mathbf{P}}(x,y) = x \cdot y \qquad S_{\mathbf{P}}(x,y) = x + y - x \cdot y$$

$$T_{\mathbf{L}}(x,y) = \max(x+y-1,0) \qquad S_{\mathbf{L}}(x,y) = \min(x+y,1)$$

$$T_{\mathbf{D}}(x,y) = \begin{cases} x & \text{if } y = 1 \\ y & \text{if } x = 1 \\ 0 & \text{otherwise} \end{cases} \qquad S_{\mathbf{D}}(x,y) = \begin{cases} x & \text{if } y = 0 \\ y & \text{if } x = 0 \\ 1 & \text{otherwise} \end{cases}$$

Figures 3 and 5, and 4 and 6, respectively, are exemplifying the first two mentioned t-norms and t-conorms.

Figure 3. The T_M t-norm

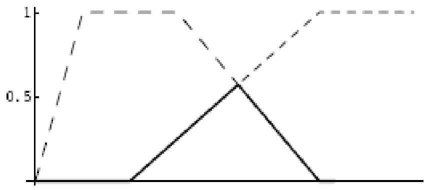


Figure 4. The S_M t-conorm

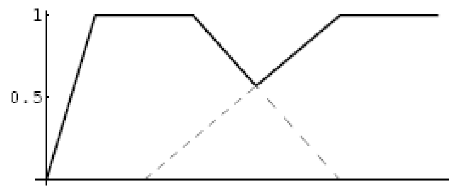


Figure 5. The T_P t-norm

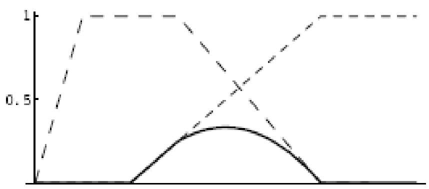
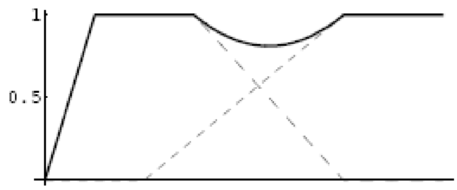


Figure 6. The S_P t-conorm



In the steps c) and d), the conclusions for each rule, as well as the general conclusion of all the rules are established. This implies an inference procedure. The basic idea of the inference procedure is that the rule that has the highest truth value will influence the output more.

There are two fundamental approaches for the inference procedure: the deductive interpretation and the assignment interpretation. The result will be one global fuzzy set. The most used in practice is the assignment interpretation, and the most common within the mentioned category is the Max-Min inference, known as Mamdani/Assilian inference, which is based on T_M t-norm and S_M t-conorm.

The final step, e), represents the defuzzyfication procedure, which actually converts the fuzzy fact into a fix output value. Three methods of realizing that are: the MON (mean of maximum), COG (center of gravity) and COA (center of area).

General issues to be taken into consideration before designing a system based on fuzzy logic are:

- an expert is always needed in the design phase;
- using fewer rules it is easier to understand the system behavior;
- it is not necessary to implement all the possible rules;
- the system may be tuned by modifying the membership functions.

IV. Research assumptions

The authors of this article have created 6 fuzzy logic models in order to check for the influence of the following aspects on the quality of the forecast:

- the form of presented financial ratios (static versus dynamic approach),
- the ability of fuzzy logic system to predict bankruptcy of companies for one year, two years and three years before,
- the proportion of bankrupt and nonbankrupt companies in testing dataset.

To conduct this research authors have used financial statements of 132 Polish stock equity companies (107 nonbankrupt and 25 bankrupt) from the years 1999-2005. There were about 230 companies quoted at Warsaw Stock Exchange Market between 1999 and 2005. That is why the population of companies taken into research contained almost all firms from all types of production and service sectors of the Polish economy. The authors excluded only the companies in the financial sector and the firms for which it was not possible to get all the financial data. Such a population of companies makes the conducted research very comprehensive, and the conclusions can be used in the evaluation of Polish stock exchange companies in the future. This population of firms was divided into:

- testing dataset "one" which was used to test models in condition of equal proportion of bankrupt and nonbankrupt firms. There were 29 "healthy" firms and 25 companies in danger of going bankrupt;
- testing dataset "two" that consisted all the companies from testing dataset "one" and additionally 78 nonbankrupt companies. This allowed to test for the effectiveness of created models to recognize the bankrupt companies among nonbankrupt firms in proportion of 19/81% ("25 bad enterprises"/"107 good ones").

All models were tested by testing dataset "one" and "two" for all three years prior to bankruptcy.

All companies were described by calculated 14 financial ratios for four years before bankruptcy and their dynamics between these years. These ratios are presented in Table 1. Additionally all firms were marked by 0-1 variable (0-bankrupt, 1-nonbankrupt).

Table 1

Financial ratios used in the research

Symbol of ratio	Type of ratio and calculation formula
PROFITABILITY RATIOS	
X1	Profit from sales / total assets
X2	Operating profit / revenues from sales
LIQUIDITY RATIOS	
X3	Current assets / short term liabilities
X4	[Current assets - inventories] / short term liabilities
X5	Working capital / total assets
DEBT RATIOS	
X6	Short term liabilities / total assets
X7	Equity / total credits
X8	(net profit + amortization) / Long term and short term liabilities
X10	Gross profit / short term liabilities
X11	(Stockholders equity + long term liabilities) / fixed assets
ACTIVITY RATIOS	
X9	Operating costs / short term liabilities
X12	Net revenues / total assets
X13	Net revenues / short term receivables
OTHER RATIOS	
X14	Log of total assets

Source: Own study.

All models were evaluated based on two types of errors and overall effectiveness:

- I type of error - $E_1 = D_1 / BR * 100\%$, where D_1 – number of bankrupt firms classified by model as nonbankrupt firms , BR – number of bankrupt companies in testing set;

- II type of error - $E_2 = D_2$ / NBR * 100%, where D_2 – number of nonbankrupt companies classified by model as a bankrupt company, NBR – number of nonbankrupt companies in testing set;
- Overall effectiveness – S = {1 – [($D_1 + D_2$) / (BR + NBR)]} * 100%.

In order to realize the assumed goals of this study, authors have used two research approaches (for each approach three fuzzy logic models were created – from one to three years before bankruptcy):

- in the first approach (static), authors have conducted a correlation analysis of all ratios from Table 1. In this approach only ratios that were highly correlated with the score (variable 0-1) and at the same time low correlated with each other were taken to the model as the entry data. The following ratios were taken into analysis:
  - one year before: X3, X8, X9, X10
  - two years before: X1, X3, X5, X7, X8
  - three years before: X1, X3, X8, X9, X10
- in the second approach (dynamic), authors have used:
  - one year before – all four ratios used in the first approach plus dynamics of X3 between first and second year prior to bankruptcy,
  - two years before - X1, X3, X7, X8 and the dynamics of X3 between second and third year prior to failure,
  - three years before - X1, X3, X8, X9 and the dynamics of X9 between third and fourth year before bankruptcy.

The idea of using the dynamic approach was that in some situations even if the individual financial ratio has a bad value, in the case when such a ratio is dynamically improving it can mean that the company is not in the process of bankruptcy, but just opposite – in the recovery process.

In order to set the critical values for membership functions in the models, authors have calculated for all ratios the first and the third quartile and median value separately for "good" and "bad" companies. The value of the third quartile of the "bad" firms was used as the threshold value for membership functions. In the next section, authors present in detail characteristics of the created models (membership functions, implemented criteria, etc.).

V. Fuzzy logic models

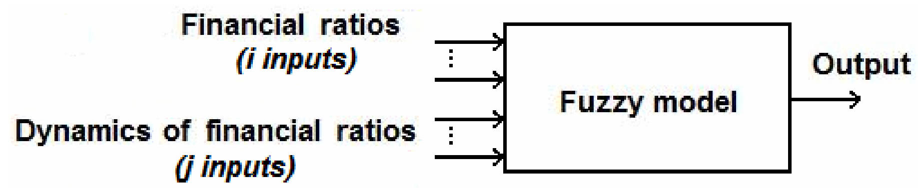
The fuzzy logic theory can be applied to many basic domains: in sociology (Bates and Young, 2003), in medicine (Kron, 2008), and in finance. The fuzzy logic decision based system developed in this paper is able to predict bankruptcy.

The basic structure of the fuzzy logic decision based system implemented in this paper is illustrated in Figure 7 (where $i \in \{4,5\}$ and $j \in \{1,0\}$).

A total of 6 scenarios were considered and a corresponding fuzzy logic decision based system was conceived for each one. Practically, 6 fuzzy models are developed.

Figure 7

The fuzzy logic decision based system



The fuzzy decision systems are based on the same principle (meaning the same types of membership functions, fuzzy conjunctions and disjunctions, inference and defuzzyfication) but they are designed to predict the possible bankruptcy for companies 1, 2 and 3 years ahead, analyzing one situation where all the inputs are represented by financial ratios and one situation where the dynamics of one financial ratio is replacing one of the inputs. Therefore the 6 fuzzy logic decision based systems will be called:

a_{11}) 1 year before without dynamics	a_{12}) 1 year before with dynamics
b_{11}) 2 years before without dynamics	b_{12}) 2 years before with dynamics
c_{11}) 3 years before without dynamics	c_{12}) 3 years before with dynamics

Considering Figure 7, for a_{11}) i=4 and j=0, for b_{11}) and c_{11}) i=5 and j=0, and for a_{12}), b_{12}), c_{12}) i=4 and j=1.

The previous chapter explained the reason and the purpose for choosing the specific ratios for each analyzed scenario. The chosen ratios will be the inputs of the fuzzy decision based system. The inputs (ratios) are associated with two membership functions. The chosen types of membership functions were the S-shaped and Z-shaped functions. The well known S-shaped and the Z-shaped membership functions are represented in (1), respectively in (2).

$$f(x;a,b)=\begin{Bmatrix} 0, & x\le a \\ 2\left(\dfrac{x-a}{b-a}\right)^2, & a\le x\le \dfrac{a+b}{2} \\ 1-2\left(\dfrac{x-b}{b-a}\right)^2, & \dfrac{a+b}{2}\le x\le b \\ 1, & x\ge b \end{Bmatrix} \quad (1) \qquad f(x;a,b)=\begin{Bmatrix} 1, & x\le a \\ 1-2\left(\dfrac{x-a}{b-a}\right)^2, & a\le x\le \dfrac{a+b}{2} \\ 2\left(\dfrac{x-b}{b-a}\right)^2, & \dfrac{a+b}{2}\le x\le b \\ 0, & x\ge b \end{Bmatrix} \quad (2)$$

The used ranges in the membership functions (in which the provided output will be between the two fixed values, the [a, b] interval in formula (2) and (3)) for a complete transition from “true” to “false” of “false” to “true” values was set after the expert knowledge accumulated by studying the behavior of numerous companies. It has been noticed that some ratios have a higher order variation and therefore a larger range will correspond to them, and some have smaller order variations and in consequence smaller range for not providing one of the two fix values.

Without describing all the used membership functions, the following examples will be pointed out: for the X3 ratio in the a_{11}, one year before study (Figure 8), the X7 ratio in the b_{12}, two years before study (Figure 9) and the X3_dyn ratio (the X3 ratio the dynamics of X3 between third and fourth year prior failure) in the c_{12}, three years before study (Figure 10). These figures are also pointing out the ranges that are used for each mentioned membership function. Each figure illustrates the two membership functions corresponding to the input variable, that is one of them is selected and some details are presented about it.

Figure 8

The membership functions for the X3 ratio in the a_{11} study

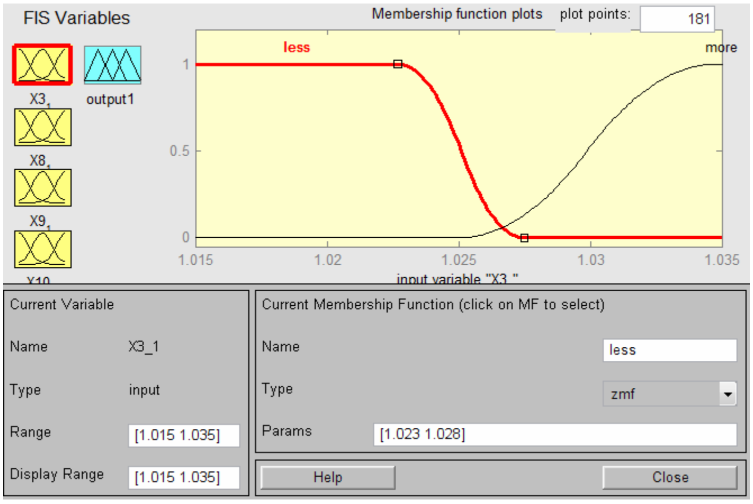


Figure 9

The membership functions for the X7 ratio in the b_{12} study

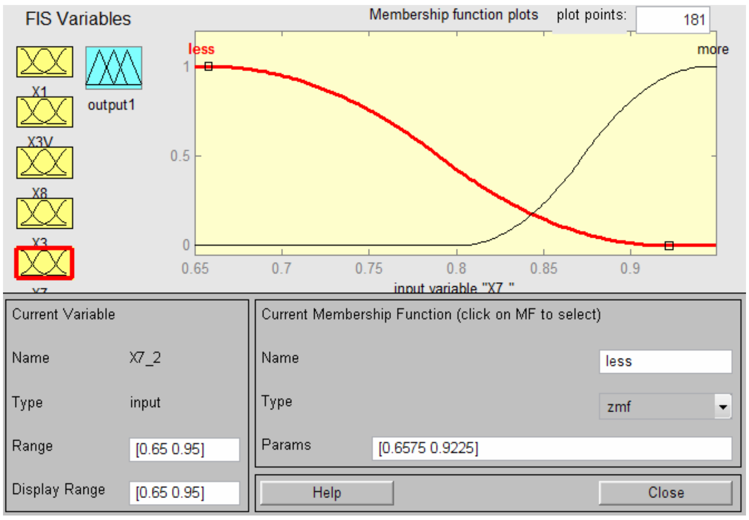
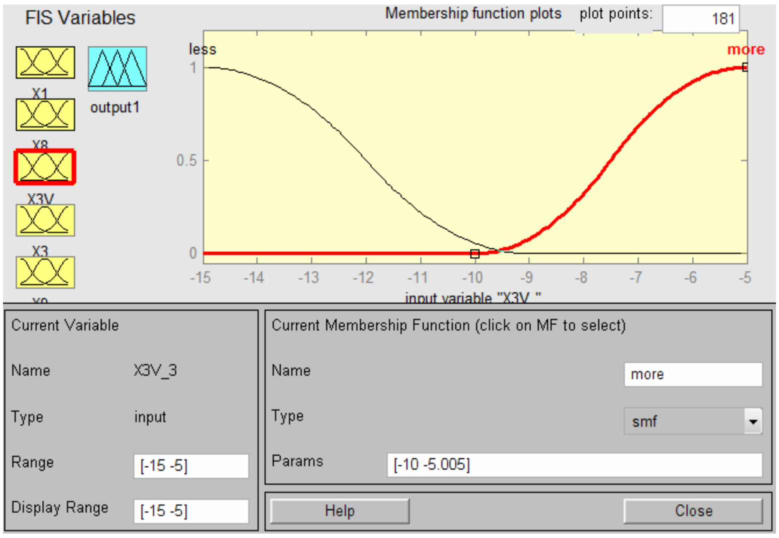


Figure 10

The membership functions for the X3_dyn in the c_{12} study



The set of rules used by the fuzzy decision system is containing 16 rules for the a11 decision based system (this one having 4 inputs) and 25 rules for the other five fuzzy systems (these ones having 5 inputs). It has to be noticed that with only 25 rules significant results will be obtained regarding the fact that the total number of possible rules is 32. As an example, from the 25 rules of the b11 fuzzy logic decision based system, 15 rules are illustrated in Figure 11.

Figure 11

A part of the set of rules used in the b_{11} fuzzy system

1. If (X1_2 is less) and (X5_2 is less) and (X8_2 is less) and (X3_2 is less) and (X7_2 is less) then (output1 is 0) (1)
2. If (X1_2 is less) and (X5_2 is less) and (X8_2 is less) and (X3_2 is less) and (X7_2 is more) then (output1 is 0) (1)
3. If (X1_2 is less) and (X5_2 is less) and (X8_2 is less) and (X3_2 is more) and (X7_2 is more) then (output1 is 0) (1)
4. If (X1_2 is less) and (X5_2 is less) and (X8_2 is more) and (X3_2 is more) and (X7_2 is more) then (output1 is 1) (1)
5. If (X1_2 is less) and (X5_2 is more) and (X8_2 is more) and (X3_2 is more) and (X7_2 is more) then (output1 is 1) (1)
6. If (X1_2 is more) and (X5_2 is more) and (X8_2 is more) and (X3_2 is more) and (X7_2 is more) then (output1 is 1) (1)
7. If (X1_2 is less) and (X5_2 is less) and (X8_2 is more) and (X3_2 is less) and (X7_2 is more) then (output1 is 0) (1)
8. If (X1_2 is less) and (X5_2 is more) and (X8_2 is less) and (X3_2 is less) and (X7_2 is more) then (output1 is 0) (1)
9. If (X1_2 is more) and (X5_2 is less) and (X8_2 is less) and (X3_2 is less) and (X7_2 is more) then (output1 is 0) (1)
10. If (X1_2 is less) and (X5_2 is less) and (X8_2 is less) and (X3_2 is more) and (X7_2 is less) then (output1 is 0) (1)
11. If (X1_2 is less) and (X5_2 is less) and (X8_2 is more) and (X3_2 is less) and (X7_2 is less) then (output1 is 0) (1)
12. If (X1_2 is less) and (X5_2 is more) and (X8_2 is less) and (X3_2 is less) and (X7_2 is less) then (output1 is 0) (1)
13. If (X1_2 is more) and (X5_2 is less) and (X8_2 is less) and (X3_2 is less) and (X7_2 is less) then (output1 is 0) (1)
14. If (X1_2 is less) and (X5_2 is more) and (X8_2 is more) and (X3_2 is more) and (X7_2 is less) then (output1 is 1) (1)
15. If (X1_2 is less) and (X5_2 is less) and (X8_2 is more) and (X3_2 is more) and (X7_2 is less) then (output1 is 0) (1)

The fuzzy conjunctions and disjunctions in all the 6 fuzzy logic decision based systems are the T_M t-norm and the S_M t-conorm (see Chapter III).

All the 6 fuzzy logic decision based systems are using the Max-Min inference (Mamdani/Assilian inference) and for defuzzyfication, the COG (center of gravity) method is applied.

VI. Comparative performance analysis of the created models

At the end of the above research, authors have conducted a comparative analysis between created fuzzy logic models in both approaches. The results of that analysis are shown in Table 2 (the numbers in brackets represent the number of companies that were classified wrongly).

Table 2

The effectiveness of the created fuzzy logic models for both approaches

Time	Effectiveness	Fuzzy logic models			
		Testing dataset "ONE"		Testing dataset "TWO"	
		First approach (static)	Second approach (dynamic)	First approach (static)	Second approach (dynamic)
1 year before	E1	16% (4)	16% (4)	16% (4)	16% (4)
	E2	10.34% (3)	6.89% (2)	28.97% (31)	19.62% (21)
	S	87.03%	88.88%	73.48%	81.06%
2 years before	E1	8% (2)	8% (2)	8% (2)	8% (2)
	E2	24.13% (7)	24.13% (7)	40.18% (43)	40.18% (43)
	S	83.33%	83.33%	65.91%	65.91%
3 years before	E1	24% (6)	20% (5)	24% (6)	20% (5)
	E2	27.58% (8)	17.24% (5)	47.66% (51)	39.25% (42)
	S	74.07%	81.48%	56.81%	64.39%

Source: Own study.

In the case of testing dataset "one", it can be seen that using the dynamic approach of analysis, the effectiveness of fuzzy logic models are generally better than using only the static representation of financial ratios. It is worth noticing that the biggest improvement of effectiveness of the model resulted by the use of dynamic approach was in the analysis of three years before bankruptcy (the increase in effectiveness by 7.41 percentage points). In the analysis of one year before, the dynamic model generated 1.85 percentage points better effectiveness than in case of static approach. In predicting bankruptcy two years before, both models (static and dynamic) got the same effectiveness 83.33%. Though, it is also important to notice that all models in both approaches got very high effectiveness of predictions – in the case of dynamic approach for all three years before bankruptcy, the models generated effectiveness at a level of over 81%. With the increase in the forecast period the effectiveness of fuzzy logic models decreased. But the smallest decrease in performance can be noticed also in the case of dynamic models.

In the case of testing dataset "two", models from the dynamic approach are also characterized by the higher effectiveness than models from the first approach. In the analysis of one year before, the dynamic fuzzy logic model got effectiveness of 81.06%, while the static model got 73.48%. Three years before, the difference was 64.39% versus 56.81%. When testing the models in proportion of 19% bankrupt and 81% nonbankrupt companies in the population of firms, the second type of mistakes increased significantly. The second type of mistakes, is the mistake of classifying a nonbankrupt company as a bankrupt. It is necessary to make a comment that the first type of mistakes is much more costly than the second type of errors to make. The first type of errors means that, for instance, a bank classifies a bankrupt company as a nonbankrupt company. It is also worth mentioning that, although the second type of mistakes is frequent, there is a probability that some of the nonbankrupt companies that models classified as in danger of bankruptcy at that time really were in such danger of financial failure. Authors marked companies as the bankrupt ones only in the cases when the formal application to the court for bankruptcy process was reported. It can mean that in some cases the companies were in danger of insolvency, but it was not reported.

VII. Conclusions and recommendations

The paper presented fuzzy logic-based bankruptcy decision models that are using as inputs financial ratios as well as dynamics of the financial ratios. The presented models were created in order to predict a possible future failure of a company, 1 year, 2 years and 3 years prior to the possible bankruptcy. The presented models performance proves to be satisfactory. Moreover, fuzzy logic models allow not only for financial ratios that in case of parametric models must have normal distribution. As results of the comparative tests, it is proved that the fuzzy logic model performance is better when dynamics of the financial ratios are also considered as inputs.

The expert system the fuzzy model once created is easy to be manipulated if a proper software is used. The current research presents the implementation of the fuzzy prediction system in Matlab software. In this context, the rules can be modified or additional rules can be introduced if the expert system improves, the membership functions can be changed or other membership functions can be easily added, the inputs can be replaced, respectively the inference procedure and the defuzzyfication method can be changed easily. Therefore the entire structure is flexible. As future development directions, the authors recommend detailed researches on fuzzy logic-based prediction strategies using other/additional input variables, other types of membership functions and corresponding new set of rules, in order to further increase the bankruptcy prediction performance.

References

Altman, E., 2006. Corporate Financial Distress and Bankruptcy, 3rd ed. New York: John Wiley & Sons Inc.

Aziz, M.A. Dar, H.A., 2004. Predicting Corporate Bankruptcy- Wither do We Stand? Loughborough University: Department of Economics, Great Britain. Available at: <http://gnu.univ.gda.pl/~eefs/pap/aziz.doc>.

Balcaen, S. Ooghe, H., 2006. 35 years of studies on business failure: an overview of the classic statistical methodologies and their related problems. *British Accounting Review*, 38, pp.63–93.

Bates, J. Young, M., 2003. Applying Fuzzy Logic to Medical Decision Making in the Intensive Care Unit. *American Journal of Respiratory and Critical Care Medicine*, 167, pp.948-952.

Chena, H.J. Huangb, S.Y. Lin, C.S., 2009. Alternative diagnosis of corporate bankruptcy: A neuro fuzzy approach. *Expert Systems with Applications*, 36(4), pp.7710-7720.

Dahiya, S. Klapper, L. , 2007. Who surives? A cross-country comparison. *Journal of Financial Stability*, 3.

Daianu, D. Lungu, L., 2008. Why is the financial crisis occurring? How to respond to it?. *Romanian Journal of Economic Forecasting*, 9(4), pp. 59-87.

Dyrberg, A., 2005. A comparative analysis of the determinants of financial distress in French, Italian and Spanish firms. Working Papers, Danmarks Nationalbank, nr 26.

Korol, T., 2005. Business bankruptcies and the use of artificial neural networks. *CeDeWu Publication Group*, Warsaw.

Korol, T., 2011.: Multi-Criteria Early Warning System Against Enterprise Bankruptcy Risk. *International Research Journal of Finance and Economics*, 61, pp.141-154.

Korol, T., Korodi, A., 2010. Predicting Bankruptcy with the Use of Macroeconomic Variables. *Journal of Economic Computation and Economic Cybernetics Studies and Research*, 1, pp. 201-220.

Kron, T., 2008. Fuzzy-logic for sociology. *Fuzzy Information Processing Society*, pp. 1-5, New York.

Morariu, N. Iancu, E. and Vlad, S., 2009. A neural network model for time-series forecasting. *Romanian Journal of Economic Forecasting*, 12(4), pp.213-223.

Moulton, W. Thomas H., 1996. Business failure pathways – environmental stress and organizational response. *Journal of Management*, 2(4).

Nastac, I. Dobrescu, E. and Pelinescu, E., 2007. Neuro-adaptive model for financial forecasting. *Romanian Journal of Economic Forecasting*, 8(3), pp.19-41.

Ooghe, H. Prijcker, S., 2006. Failure processes and causes of company bankruptcy – a typology. *Working Paper of Universiteit Gent*.

Purvinis, O. Virbickaite,˙R. and Sukys P., 2008.: Interpretable Nonlinear Model for Enterprise Bankruptcy Prediction. *Nonlinear Analysis: Modeling and Control,* 13(1), pp.61–70.

Richardson, B. Nwankwo, S. and Richardson S., 1994. Understanding the causes of business failure crises. *Management Decision Journal*, 32(4).

Sharda, R. Wilson, L., 1994. Bankruptcy prediction using neural networks. *Decision Support Systems*, nr 11, pp.548,

Spanos, M. Dounias, G. Matsatsinis, N. and Zopounidis, C., 1999. A Fuzzy Knowledge-Based Decision Aiding Method for the Assessment of Financial Risks: The Case of Corporate Bankruptcy Prediction. *European Symposium on Intelligent Techniques*, Crete, Greece.

Thomaidis, N. Gounias, G.D. and Zopounidis C.D., 1999. A fuzzy rule based learning method for corporate bankruptcy prediction. *ACAI 99*, Chania, Greece.