9. IS THE ROMANIAN BUSINESS CYCLE CHARACTERIZED BY CHAOS?[1]

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Abstract

I compare the two alternative paradigms of business cycles for the case of the Romanian economy, namely the mainstream view that business cycles are driven by stochastic shocks and the nonlinear view, known as the endogenous business cycles theory, which states that business cycles are driven by deterministic processes. The comparison is based on the run of several tests for nonlinearity and chaos, such as the BDS test or the maximum Lyapunov exponent.

Keywords: business cycles, nonlinear analysis, chaos, endogenous business cycles

JEL Classification: C13, C22, E32

1. Introduction

The mainstream macroeconomic model, known as the dynamic stochastic general equilibrium model, is a development of the real business cycles (RBC, hereafter) as proposed by Kydland and Prescott (1982). Originally, the model was based on the assumption of perfect competition, with the business cycles viewed as driven by stochastic shocks. The following extensions of the baseline model, which took into account the various imperfections that are present in the real economies, preserved, however, several fundamental assumptions specific to the RBC: rational expectations of the agents, optimizing agents (firms and households), with the economy viewed as driven by stochastic exogenous shocks to technology (Solow residual), preferences, policy rules, etc.

An alternative view of the business cycles dates back to the first years of macroeconomics. Models such as those proposed by John Hick, Nicholas Kaldor or John Goodwin, advocated a view based on the hypothesis that the model economy is self-generating recurrent cycles that result from the structure of the underlying process (see Benhabib, 1994). This second view was for a long period considered as not very

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relevant. However, after the '80s, the interest in this view was reignited. Among the factors that contributed to this are the contributions by Grandmont (1985) and also the rapid development of nonlinear tests, mainly derived from physics.

Starting with the '80s, a series of contributions, such as those of Brock (1988) or Scheinkman (1990), discovered that it was evidence that some economic time series might be characterized by chaos. However, the findings remain debated and more recent research points that no definite conclusion are reached yet.

Most of the research on the existence of chaos was done on financial data or exchange rates. There is also some research done on macroeconomic data (see Takala and Viren, 1996) although this is rather a marginal interest, mainly due to the limitations of data for the case of macroeconomic time series.

In this context, the present paper proposes to test for the existence of deterministic chaos in the Romanian business cycles. Some previous paper, such as those of Albu (2001) or Purica and Caraiani (2009), underscored some of the nonlinear features of business cycles in the Romanian economy. In this study, we go deeper into the nonlinear features of the Romanian business cycles by testing for the existence not only of nonlinear properties but also of chaotic patterns in the data.

The paper is organized as follows. The second section discusses the methodology and explains the techniques used. The following section shows the estimation results and discusses the main economic implications of the results. The last section concludes and suggests some possible extensions of the present paper.

2. Methodology

The methodology is based on a series of tests and techniques derived from nonlinear techniques originally used in physics. We focus on the discussion of three main techniques and tests, which were extensively employed in applications in economics and finance: the BDS test, the Hurst coefficient and the maximum Lyapunov exponent.

2.1. The BDS Test

The BDS test is due to Brock, Dechert and Scheinkman (1987), who proposed a way to test whether a series is independent and identically distributed or not. We present the statistics in a short manner, following Zivot and Wang (2005) or Tsay (2010). Formally, the BDS statistics is defined as:

$$x_t^m = \left(x_t, \ldots, x_{t-m+1}\right)$$

with: *T* the number of observations, *δ* the distance between two points and $\sigma_k(\delta,T)$ obtained from $\sigma_k(\delta)$ when *C* and *N are* replaced by $C_1(\delta,T)$ and $N(\delta,T)$. Key to this statistics is the concept of correlation integral, with $C_k(\delta,T)$ being the correlation integral for a *k* embedding dimension. Through the correlation integral one estimates the probability that any of two *k*-dimensional points are within a δ distance of each other.

The statistics: $c_{m,\varepsilon} = \frac{2}{T_m(T_m-1)} \sum\sum_{m \le s \le t \le T} I(x_t^m, x_s^m; \varepsilon)$ is asymptotically distributed as normal, with mean zero and variance $T_m = T - m + 1$.

Under the null hypothesis of independent and identically distributed series, the following relationship holds:

$$C_{1,\varepsilon}^m = \Pr(|x_t - x_s| < \varepsilon)^m,$$

for $V_{m,\varepsilon} = \sqrt{T} \frac{c_{m,\varepsilon} - c_{1,\varepsilon}^m}{s_{m,\varepsilon}}$ for *k* and *δ* that are fixed.

The BDS statistics converges in distribution to *N(0,1):*

$$V_{m,\varepsilon} \xrightarrow{d} N(0,1)$$

Thus, the null hypothesis of an independent and identical distributed series is rejected at 5% significance level each time $|V_{m,\varepsilon}| > 1.96$.

2.2. The Hurst Coefficient

The Hurst coefficient dates back to the contributions of British hydrologist H. Hurst who developed this approach when studying the flows of the river Nile. He proposed the R/S method to estimate the level of what is known today as the Hurst coefficient.

We present in a short manner what is meant by the Hurst coefficient (see Gao *et al*., 2007 for a more in-depth analysis and presentation of different approaches in computing it).

Hurst (1951) found that many time series behave according to the following power-law relationship:

$$\left[\frac{R(\tau)}{S(\tau)}\right] = \left(\frac{\tau}{2}\right)^H$$

where: *H* is the Hurst coefficient.

For discrete time series analysis, the procedure consists in the following steps. Consider a time series y_n, n=1,…,N. The running sum y_m of the time series is given by:

$$y_N = \sum_{i=1}^{N} (y_i - \bar{y}_N)$$

The range can be defined as:

$$R_N = (y_N)_{\max} - (y_N)_{\min}$$

With

$$S_N = \sigma_N$$

As we are interested how (R/S) behaves when, for successive subintervals of τ of N, we substitute τ for N and get the Hurst coefficient from the following equation:

$$E\left[\frac{R_N}{S_N}\right] \sim CN^H$$

The value of the Hurst exponent helps us to distinguish between different types of time series

- A series for which 0<H<1/2 is considered as behaving in an anti-persistent way;
- The time series for which 1/2<H<1 are characterized by persistent correlation;
- Finally, for the series for which the Hurst coefficient is equal to ½, there is short range dependency.

2.3. The Maximum Lyapunov exponent

As the literature points (see Dingwell, 2006), there is no unanimously accepted definition of chaos. However, some authors pointed to some workable definition. For example, Strogatz (1994) showed that chaos can be defined as "aperiodic long-term behavior in a deterministic system that exhibits sensitive dependence on initial conditions". The Lyapunov exponent is used in order to detect the existence of the sensitivity to the initial conditions.

We describe in a succinct manner the mathematical foundations, following Dingwell (2006). Since the data used is discrete, the approach presented here is the one based on discrete systems.

We consider a map of the following form:

$$\Lambda = \left\langle \ln \left\langle \left\| V_{i+k} - V_{j+k} \right\| \right\rangle_{mean_j} \right\rangle_{mean_i}$$

where: x is a vector of state variable that can be single or multidimensional, μ is a vector of parameters, while k is an index indicating the iteration step.

The baseline idea of the Lyapunov exponent is to trace two trajectories of the system that start from two different initial states. Formally, one can write:

$$\lambda \approx \frac{1}{n}\ln\left|\frac{\varepsilon_n}{\varepsilon_0}\right| = \frac{1}{n}\ln\left|\frac{f^n(x_0+\varepsilon_0) - f^n(x_0)}{\varepsilon_0}\right|$$

Here λ is the Lyapunov exponent, ε_n is the distance between the two trajectories after n iterations, with the assumption that:

$$\Lambda(k) \sim k$$

Since we are interested in the case when ε_0 tends towards zero, the final expression is given by:

$$\lambda = \lim_{n\to\infty}\left\{\frac{1}{n}\sum_{i=0}^{n-1}\ln\left(f'(x_i)\right)\right\}$$

There are many methods proposed to estimate the Lyapunov exponent, among them those proposed by Wolf *et al*. (1985), Abarbanel *et al*. (1990), Kantz and Schreiber (2004) or Rosenstein *et al*. (1993). We will apply a few of these techniques in the estimation section.

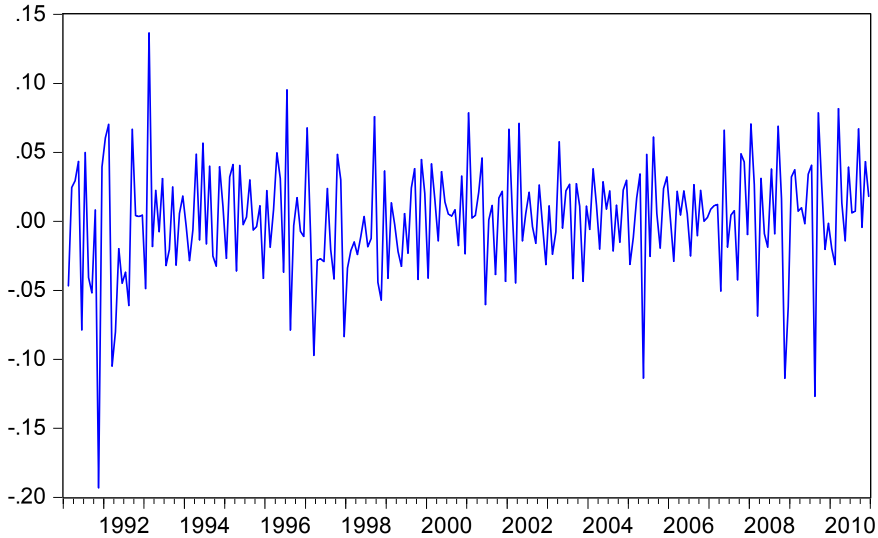
3. Data and results

3.1. Data used

I use monthly data of the Romanian index of industrial production. The data dates back to January 1991 and the sample ends up in December 2010. The initial series is not seasonally adjusted and is taken as a series with December 1990 as fixed base. Before using it, the series was seasonally adjusted using the Census X12 procedure. Afterwards, the series was logged using the natural logarithm and then transformed by means of the first difference. The final series, see Figure 1, is stationary.

Figure 1

Log differenced series of the index of industrial production



Source: Own Computations.

3.2. The BDS Test

In the literature (see, for example, Barnet *et al.*, 1995), the BDS test is viewed as a first step in testing for nonlinearity. While BDS cannot be conceived as a test of linearity versus chaos, it is best to be viewed as a test of linearity against a general class of possible nonlinear models.

In Table 1 we present the results of the BDS test for the series of industrial production presented in the previous section. We consider embedding dimensions from 2 to 7 as well as a radius that depends on the standard deviation, with four different cases considered. Overall, a wide range of possibilities is covered through this approach. We can see that the null hypothesis of linearity is rejected for the cases considered. What kind of nonlinearity characterizes the series could, however, be revealed with the help of different tests, as resulting from the following sections.

Table 1

The BDS results

σ/m	0.5σ	σ	1.5σ	2σ
2	4.1095	3.9243	3.4019	2.3396
3	6.4171	5.3689	4.2113	2.6029
4	7.2515	5.8589	4.4819	2.5185
5	8.4400	6.0996	4.6433	2.5984
6	8.4870	6.1387	4.6742	2.7095
7	9.5266	6.5865	4.7880	2.8108

Source: Own computations.

3.3. Estimated Maximum Lyapunov Exponent

Computing the maximum Lyapunov exponent is one of the most direct way to test for the existence of chaos in a given time series. At the time, the economic time series, and especially the macroeconomic ones, are not as long as the series in physics, where these concepts emerged from. In order to account for the possibility that the results are sensitive to the method employed in deriving the maximum Lyapunov exponent, we use several different approaches found in literature.

We consider two of the most used tests when computing the maximum Lyapunov exponent. Both tests are not only known to perform well for chaotic process, but can also deal in a robust manner with the presence of noise, which characterizes most of the economic and financial time series. The key parameter of embedding dimension is set by having in mind three different cases, with dimensions that vary from 1 to 7, from 2 to 7 or from 3 to 7. The results are presented in Table 2. We obtain clear evidence of chaotic dynamics in the series of industrial production. Some further tests could take into consideration also the low number of observations as well as how much the noise distorts the computation of the Lyapunov exponent.

Table 2

The Maximum Lyapunov Exponent

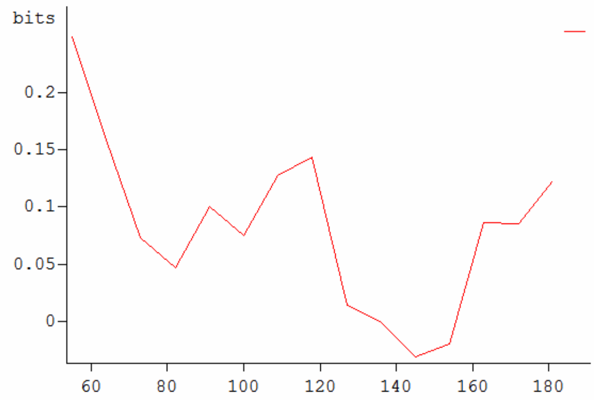
Embedding Dimension	Kantz (1994) approach	Rosenstein et al. (1993) approach
1:7	0.01125	0.00691
2:7	0.01435	0.00632
3:7	0.01876	0.01876

Source: Own computations.

As a final assessment of the Lyapunov exponent, we also derive a moving window maximum Lyapunov exponent. For a sliding window of 110 observations, with a step of 15, the local maximum Lyapunov exponent is computed using the Wolf approach.

Figure 2

Moving Window Maximum Lyapunov Exponent



Source: Own computations.

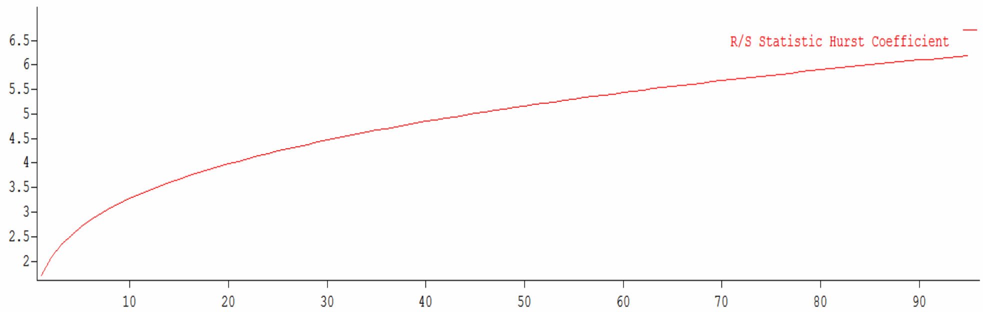
The moving Lyapunov exponent suggested can be understood from the perspective of the dynamics of Romanian economy. The highest Lyapunov exponent is registered at the beginning of transition, when the economy was affected by a sudden drop in output, which can be rather interpreted within the endogenous business cycle theory. It is also remarkable that the other two recessions were characterized by rising Lyapunov exponents as compared to the growth period. Moreover, the only period with a negative Lyapunov exponent was registered within the stable growth period between 2000 and 2010. Nevertheless, much caution should be exerted, especially towards this last graphic, as it is based on less desirable number of observations.

3.4. The Hurst Coefficient

First, we compute the coefficient based on the R/S approach (see Figure 3). The Hurst exponent, calculated from the R/S statistics, is 0.283276.

Figure 3

Estimated Hurst Coefficient based on R/S Statistics



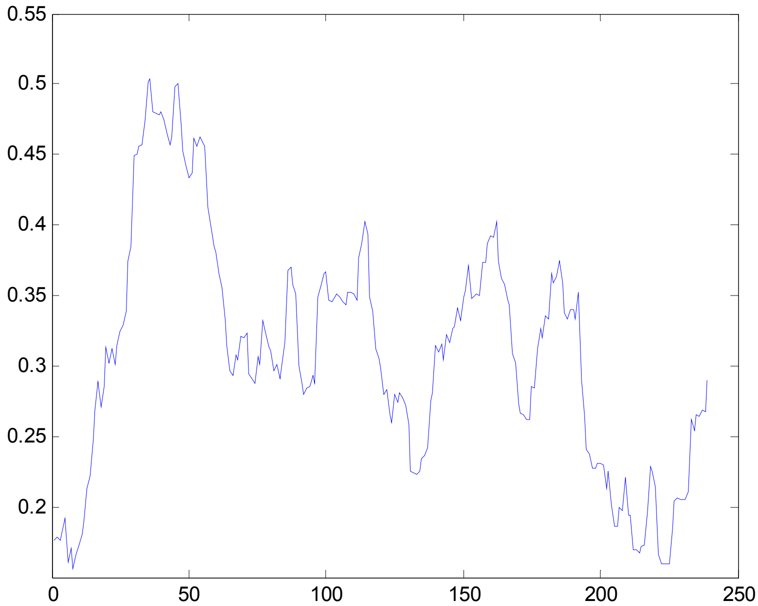
Source: Own computations.

This is an indication that the series is characterized by long-memory and, moreover, is also characterized by anti-persistence.

Given the sensitivity of the value of the Hurst coefficient to the methods used, I also employ here a rolling estimation of the Hurst coefficient. Such an approach not only gives a more complete picture of the coefficient, but allows for the identification of specific episodes where the value of the Hurst coefficient fluctuated. The rolling estimation is based on rolling windows. These are windows of a fixed dimension. The values on the horizontal axis stand for the starting point of the window (the first observation).

Figure 4

Estimated rolling Hurst coefficient



Source: Own computations.

Overall, the Hurst coefficient is found to range between 0 and 0.5 indicating an anti-persistent time series. Furthermore, the value of the Hurst coefficient dropped consistently during the two periods of recessions, as one may see from the period corresponding to the beginning of the sample, when the transformational recession happened, from the period corresponding to 1997-1998, as well as from the last period, after observation 200, that corresponds roughly to the period after the Fall of 2008, when the last recession in Romania began.

4. Conclusion

The recent financial crisis highlighted the limits of our understanding of the business cycles. Coupled with the findings that numerous time series have nonlinear or even chaotic properties, the view of the economy as driven by inherently unstable dynamic processes, as supported by the endogenous business cycle theory, has received increased support.

This paper adds evidence to the case of the presence of nonlinearities and chaotic patterns in the macroeconomic time series. Although limited due to the nature of macroeconomic data, which is also the case for financial data, relative to the size of data from sciences such as physics, geology or biology, the paper found solid evidence in the favor of nonlinearity and deterministic chaos in the macroeconomic fluctuations in Romania.

More work could be done in this direction by testing and modeling the nature of processes driving the Romanian business cycles, which could improve the forecasts capabilities.

References

Abarbanel, H.D. 1996. *Analysis of Observed Chaotic Data*. New York: Springer Verlag.

Albu, L.L. 2001. Evolution of Inflation-Unemployment Relationship in the Perspective of Romania's Accession to EU. *Romanian Journal of Economic Forecasting, 2(3-4): 5-23.*

Barnett, W. et. al.. 1995. Robustness of Nonlinearity and Chaos Tests to Measurement Error, Inference Method and Sample Size. *Journal of Economic Behavior and Organization, 27, pp. 301-320*.

Benhabib, J. 1994. *Cycles and Chaos in Economic Equilibrium*. Princeton, MA: Princeton University Press.

Brock, W.A. Dechert, W.D. and Scheinkman, J.A., 1987. A Test for Independence Based on the Correlation Dimension, mimeo.

Brock, W., 1988. Nonlinearity and Complex Dynamics in Economics and Finance. In P. Anderson K. Arrow and D. Pines (Eds.), *The Economy as an Evolving Complex System*. New York: Addison Wesley.

Dingwell, J. 2006. Lyapunov Exponents. In Metin Akay *(Eds.). Wiley Encyclopedia of Biomedical Engineering*. John Wiley & Sons, Inc.

Gao, J. Cao, Y. Tung, W. and Hu, J., 2007. *Multiscale Analysis of Complex Time Series. Integration of Chaos and Random Fractal Theory, and Beyond*. New Jersey: John Wiley and Sons.

Grandmont, J.M. 1985. On Endogenous Competitive Business Cycles. *Econometrica 53*, pp. 995-1045.

Hurst, H. 1951. Long Term Storage Capacity of Reservoirs. *Transactions of the American Society of Civil Engineers 116*, pp. 770-808.

Kantz, H. 1944. A Robust Method to Estimate the Maximal Lyapunov Exponent for a Time Series. *Physical Letter A,* 185 (1), pp. 77-87.

Kydland, F. and Ed. Prescott. 1982. Time to Build and Aggregate Fluctuations. *Econometrica*, 50(6), pp. 1345-1370.

Mandelbrot, B.B., 1966. Long-run linearity, locally Gaussian process, H-spectra and infinite variances. *International Economic Review,* 10, pp. 82-111.

Parlitz, U., 1998. Nonlinear Time Series Analysis. In: J.A.K. Suykens and J. Vandewalle, Eds.*Nonlinear Modeling - Advanced Black-Box Techniques.* Kluwer Academic Publishers, Boston, 209-239.

Purica, I. and Caraiani, P., 2009. Second order dynamics of economic cycles. *Romanian Journal of Economic Forecasting,* 10(1), pp. 36-47.

Rosenstein, M. Collins, J. and DeLuca. C., 1993. A Practical Method for Calculating Largest Lyapunov Exponents from Small Data Sets. *Physica D: Nonlinear Phenomena 65*, pp. 117-134.

Scheinkman, J., 1990. Nonlinearities in Economic Dynamics. *Economic Journal, 100*, pp. 33-47.

Strogatz, S.H., 1994. *Nonlinear Dynamics and Chaos: With Applications to Physics, Biology, Chemistry and Engineering*. New York: Adison-Wesley.

Takala, K. and Viren, M., 1996. Chaos and nonlinear dynamics in financial and nonfinancial time series: Evidence from Finland. *European Journal of Operational Research, 93*, pp. 155-172.

Tsay, R., 2010. *Analysis of Financial Time Series*. John Wiley and Sons.

Wolf, A. Swift, J. Swinney, H. and Vastano, J., 1985. Determining Lyapunov Exponents from a Time Series. *Physica D: Nonlinear Phenomena, 16(3)*, pp. 285-317.

Zivot, E. and Wang, J., 2005. *Modeling Financial Time Series with S-PLUS*. New York: Springer Verlag.