2. Statistical Analysis of KEMIRA Type Weights Balancing Methods

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Abstract

The article analyzes Multiple Criteria Decision Making (MCDM) problem when there are two different groups of evaluating criteria. It was shown how criteria weights can be calculated according to weights balancing method by formulating optimization task. Case study of the small dimensions problem was solved by Kemeny Median Indicator Ranks Accordance (KEMIRA) method with options re-selection. Next, 8 various candidates sorting algorithms – 6 based on voting theory methods and 2 algorithms based on Kemeny median – were compared with each other. Monte Carlo experiments were conducted for the cases of 3-10 experts, 3-5 candidates and probability values of correct decision p=0.4-0.8. The highest percent of correct decisions and the lowest percent of failed voting procedures were demonstrated by algorithms based on Kemeny median.

Keywords: Multiple Criteria Decision Making, KEMIRA method, Kemeny median, Monte Carlo method, voting theory

JEL Classification: C15, C61, D72, D81

I. Introduction

In application of MCDM methods it is important to establish priorities of the assessment criteria, since the quality of further application of the method (criteria weighting and alternatives ranking) depends on this. Generally, criteria priorities are determined with respect to experts opinion. However, expert evaluation has shortcomings. Morselli (2015) analyzed the role of intuition in decision-making, as well as the effect of emotions, explaining the failures of human decision making processes.

A comparative analysis of group decision methods was carried out by Fishburn (1971). The paper examines some explicit social choice functions that are generalizations of

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the simple majority decision rule. Two different classes of such functions for choice from among two or more alternatives are summation procedures and completions of Condorcet's criterion (see Condorcet, 1785). A summation method of Borda (1784) and a Condorcet completion method were compared in Fishburn (1971) by computer simulation. With the number of voters varied from 3 to 21 and the number of alternatives varied from 3 to 9, about 90 percent of the 70,000 cases examined had a common winner for the two methods.

Problem of multidimensional ordinal measurement by applying the well-known Kemeny rule from social choice theory was analysed by Athanassoglou (2015) and Muravyov (2014).

The main ideas and contributions of the field of public choice, the properties of different voting rules are surveyed by Mueller (1997). A review of the theory of voting in medieval Europe is presented by McLean (1990). Three procedures for elections when there are more than two candidates were analyzed. Two of the three propose Borda methods and the third a Condorcet method of successive pairwise comparison. All three medieval works discuss problems of manipulation.

Nurmi and Meskanen (2000) have noticed that the classic voting paradoxes, viz. Borda's and Condorcet's, have obvious implications for certain MCDM situations. It implies that the notion of the best alternative, given a set of criteria and information about the ordinal ranking of the alternatives on those criteria, can be essentially arbitrary. Authors discussed the implications of paradox of multiple elections which is a situation where the result of multiple-item election may be a policy alternative that nobody voted for. Naamani-Dery *et al.* (2015) introduced novel heuristics and showed how one can operate under the Borda voting protocol.

Merlin, Tataru and Valognes (2002) calculated the probability of Condorcet's voting paradox in three-candidate elections. The probability of individual and coalitional manipulation of three specific social choice functions (Borda rule, Copeland rule, Plurality rule) in three-alternative elections was calculated by Diss (2015).

Some simple proofs of variations of Arrow's theorem (see Arrow, 1951) with very little mathematical knowledge required is presented in the course guide of Lum and Kurtz (1975). Approval voting has been investigated by Maniquet and Mongin (2015) in an Arrovian framework of collective preference and in connection with Arrow's impossibility theorem. Although many axiomatic results concerning aggregation procedures in multi-criteria decision aiding have been obtained in the framework of social choice theory, Marchant (2003) argued that social choice theory, which is helpful for a better understanding of some aggregation procedures, is not totally appropriate for multi-criteria decision aiding.

Close relation of social choice theory with multiple criteria decision-making (MCDM) especially in group decision contexts was revealed in the article by Srdjevic (2007). Author investigated two possible contexts in modeling decentralized decision problems in water management. Potthoff (2010) ascertained that public opinion polls are not generally designed to try to identify a Condorcet candidate. Modern social choice theory treats voting as a method for aggregating diverse preferences and values unlike Condorcet approach, where voting is a method for aggregating information.

The paper by Yu and Hu (2010) develops an integrated MCDM approach that combines the voting method and the fuzzy TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) method to evaluate the performance of multiple manufacturing plants in a fuzzy environment. The voting method is used in this study to determine the appropriate criteria weights. Akhavan *et al.* (2015) in their paper proposed a systematic approach for an effective strategic alliance partner selection. The results are combined with the help of the Borda method to choose the best alternative.

In the study by Kim and Chung (2013) TOPSIS combined with the voting methods (Borda count and Copeland's methods) approach was applied to a water-resource system in South Korea. Noticeable that rankings from the voting-based methods did not differ much from those from non-voting-based (i.e., average-based) methods. Madani, Read and Shalikarian (2014) employed several practical and popular voting methods to solve a multi-stakeholder hydro-environmental management problem.

In Srdjevic *et al.* (2015) it was shown how the social choice theory (SCT) with its voting systems can be efficiently combined with MCDM and AHP (Analytic Hierarchy Process) in particular, in various group-decision contexts.

Dadelo *et al.* (2014) have proposed a method of balancing criteria weights. The idea of this method is to obtain the weights while solving a conditional optimization problem formulated for that purpose. Weights were ranked in the order of criteria importance, which takes into account the opinion of experts calculating the average of ranks for each criterion. In Krylovas *et al.* (2014) Kemeny median has been proposed instead of the average of ranks and this method was named KEMIRA (KEmeny Median Indicator Ranks Accordance). The method has been modified and adapted to solve specific problems in Kosareva *et al.* (2016), Krylovas *et al.* (2016).

This article discusses the methods that we call KEMIRA type methods and compare them to each other and voting theory methods. For the best option determination in various areas voting theory methods are naturally suited. In the article 6 selected widely used voting theory methods are compared with two new Kemeny median-based methods. The statistical experiment was carried out by the Monte Carlo simulations. The average numbers of correct decisions, as well the average numbers of failed voting procedures were compared for different methods.

II. Task formulation

Suppose we have N objects evaluation results by criteria from two different criteria groups X and Y (for example, internal and external criteria):

Object	x_1	x_2	$\cdots$	x_{n_x}	y_1	y_2	$\cdots$	y_{n_y}
1	$x_1^{(1)}$	$x_2^{(1)}$	$\cdots$	$x_{n_x}^{(1)}$	$y_1^{(1)}$	$y_2^{(1)}$	$\cdots$	$y_{n_y}^{(1)}$
2	$x_1^{(2)}$	$x_2^{(2)}$		$x_{n_x}^{(2)}$	$y_1^{(2)}$	$y_2^{(2)}$		$y_{n_y}^{(2)}$
$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$
N	$x_1^{(N)}$	$x_2^{(N)}$	$\cdots$	$x_{n_x}^{(N)}$	$y_1^{(N)}$	$y_2^{(N)}$	$\cdots$	$y_{n_y}^{(N)}$

Evaluation results $x_i^{(j)}$, $y_i^{(j)}$ gain values from 0 to 1 and the best object assessment is $(X;Y)=(1,1,...,1)$, the worst evaluation – $(X;Y)=(0,0,...,0)$.

Criteria $x_i^{(j)}$ and $y_i^{(j)}$ have different importance, which is identified by expert established criteria preferences:

$$x_{i_1} \succ x_{i_2} \succ ... \succ x_{i_{n_x}} \ ; \ y_{j_1} \succ y_{j_2} \succ ... \succ y_{j_{n_y}} \ . \tag{1}$$

Our goal is to create the criterion function which enables selection of the "best" objects according to criteria groups X and Y evaluations.

Write down weighted averages

$$W_{W_x}(X)=\sum_{j=1}^{n_x} w_{xi_j} x_{i_j} \ ; \ W_{W_y}(Y)=\sum_{k=1}^{n_y} w_{yj_k} x_{j_k} \ , \tag{2}$$

were coefficients w_{xi_j}, w_{yi_k} satisfy the conditions agreed with preferences (1):

$$\begin{cases} w_{x_{i_1}} \succ w_{x_{i_2}} \succ ... \succ w_{x_{i_{n_x}}} \geq 0, \ \sum_{j=1}^{n_x} w_{xi_j} = 1, \\ w_{y_{j_1}} \succ w_{y_{j_2}} \succ ... \succ w_{y_{j_{n_y}}} \geq 0, \ \sum_{k=1}^{n_y} w_{yj_k} = 1. \end{cases} \tag{3}$$

Let's denote $X_{\alpha_x}, Y_{\alpha_y}$ subsets of the set $J=\{1,2,...,N\}$, which elements satisfy respective inequalities

$$W_{W_x}(X^{(j)}) \geq \alpha_x; \ W_{W_y}(Y^{(j)}) \geq \alpha_y, \ j \in J. \tag{4}$$

Subsequently, X_{α_x} and Y_{α_y} are the sets of "good" objects according to criteria X and Y respectively with thresholds α_x and α_y. Notice, that and $X_\alpha = Y_\alpha = \varnothing$, when $\alpha > 1$. Consider the sets

$$A = X_{\alpha_x} \cap Y_{\alpha_y}, B = \left(X_{\alpha_x} \cap Y_{\alpha_y}\right) \setminus A \tag{5}$$

and denote the number of their elements: $|A| = nA, |B| = nB.$

Therefore, the set A contains objects satisfying both criteria (4), the so-called "good" objects, and the set B - satisfying only one of the mentioned criteria (the "doubtful" objects). At higher parameters α_x and α_y values, in general, the number of elements nA of the set A decreases. Good balancing of weights w_x, w_y provides creating of the weighted averages (2), which will allow to construct the set A with sufficient number of elements nA (for example, 15 or 20% of N) and the set B containing minimum number of items nB. Optimal weights allow to distinguish the biggest set of "good" objects (according to both criteria) and the smallest set of "doubtful" objects (according to one criterion). Let's construct two weights balancing quality assessing functions (metrics):

$$W^1(w_x, w_y) = \sum_{j=1}^{N} \left| W_{w_x}(X^{(j)}) - W_{w_y}(Y^{(j)}) \right|, \tag{6}$$

$$W^2(w_x, w_y) = \sqrt{\sum_{j=1}^{N} \left(W_{w_x}(X^{(j)}) - W_{w_y}(Y^{(j)}) \right)^2} \tag{7}$$

and solve optimization problems

$$\min_{w_x, w_y} W^1(w_x, w_y),\ \min_{w_x, w_y} W^2(w_x, w_y), \tag{8}$$

when weights w_x, w_y satisfy (3). We'll maximize the agreement between two criteria groups X and Y by minimizing the values of distance functions (8).

In this article, we will not examine algorithms for solution of problems (8), (3) (see Dadelo (2014). We'll show, how problem of small dimensions n_x, n_y, N can be solved by options re-selection.

III. Example

Consider $N = 20$ objects evaluations:

Object	x_1	x_2	x_3	y_1	y_2	y_3	y_4
1	0.3	0.5	0.6	0.3	0.5	0.6	0.4
2	0.4	0.9	0.5	0.9	0.5	0.6	0.6
3	0.6	0.6	0.2	0.2	0.2	0.6	0.3
4	0.2	0.6	0.3	0.3	0.4	0.7	0.4
5	0.3	0.4	0.7	0.3	0.8	0.6	0.4
6	0.4	0.3	0.8	0.5	0.1	0.2	0.7
7	0.6	0.4	0.5	0.4	0.8	0.6	0.2
8	0.1	0.2	0.7	0.3	0.8	0.5	0.2
9	0.4	0.8	0.6	0.6	0.6	0.1	0.7
10	0.2	0.3	0.8	0.4	0.2	0.9	0.6
11	0.5	0.2	0.6	0.7	0.3	0.5	0.1
12	0.6	0.1	0.7	0.9	0.5	0.2	0.9
13	0.4	0.2	0.9	0.4	0.1	0.8	0.6
14	0.6	0.7	0.3	0.4	0.5	0.5	0.8
15	0.5	0.1	0.9	0.4	0.1	0.5	0.9

Object	x_1	x_2	x_3	y_1	y_2	y_3	y_4
16	0.5	0.2	0.9	0.4	0.8	0.3	0.2
17	0.4	0.1	0.8	0.3	0.3	0.8	0.6
18	0.6	0.4	0.5	0.2	0.2	0.9	0.4
19	0.5	0.8	0.4	0.8	0.7	0.4	0.1
20	0.1	0.5	0.9	0.9	0.8	0.3	0.3

Here $n_x = 3, n_y = 4$. Suppose, that experts assigned the following criteria priorities of the form (1):

$$x_2 \succ x_1 \succ x_3;\ y_3 \succ y_4 \succ y_2 \succ y_1. \tag{9}$$

Let W_x^i, W_y^i be non-negative integers. Make the weights w_x and w_y so:

$$\begin{gathered} w_x = \frac{1}{10}\left(W_x^1, W_x^2, W_x^3\right), W_x^2 \geq W_x^1 \geq W_x^3 \geq 0, \\ W_x^1 + W_x^2 + W_x^3 = 10, \end{gathered} \tag{10}$$

$$\begin{gathered} w_y = \frac{1}{10}\left(W_y^1, \dots, W_y^4\right), W_y^3 \geq W_y^4 \geq W_y^2 \geq W_y^1 \geq 0, \\ W_y^1 + W_y^2 + W_y^3 + W_y^4 = 10. \end{gathered} \tag{11}$$

So, there are $i = 14$ weights w_x^i combinations satisfying conditions (10):

i	1	2	3	4	5
w_x^i	(1.0,0.0,0.0)	(0.9,0.1,0.0)	(0.8,0.2,0.0)	(0.8,0.1,0.1)	(0.7,0.3,0.0)
i	6	7	8	9	10
w_x^i	(0.7,0.2,0.1)	(0.6,0.4,0.0)	(0.6,0.3,0.1)	(0.6,0.2,0.2)	(0.5,0.5,0.0)
i	11	12	13	14	
w_x^i	(0.5,0.4,0.1)	(0.5,0.3,0.2)	(0.4,0.4,0.2)	(0.4,0.3,0.3)	

and $i = 23$ weights w_y^i, satisfying conditions (11):

i	1	2	3	4
w_y^i	(1.0,0.0,0.0,0.0)	(0.9,0.1,0.0,0.0)	(0.8,0.2,0.0,0.0)	(0.8,0.1,0.1,0.0)
i	5	6	7	8
w_y^i	(0.7,0.3,0.0,0.0)	(0.7,0.2,0.1,0.0)	(0.7,0.1,0.1,0.1)	(0.6,0.4,0.0,0.0)
i	9	10	11	12
w_y^i	(0.6,0.3,0.1,0.0)	(0.6,0.2,0.2,0.0)	(0.6,0.2,0.1,0.1)	(0.5,0.5,0.0,0.0)

i	13	14	15	16
w_y^i	(0.5,0.4,0.1,0.0)	(0.5,0.3,0.2,0.0)	(0.5,0.3,0.1,0.1)	(0.5,0.2,0.2,0.1)
i	17	18	19	20
w_y^i	(0.4,0.4,0.2,0.0)	(0.4,0.4,0.1,0.1)	(0.4,0.3,0.3,0.0)	(0.4,0.3,0.2,0.1)
i	21	22	23	
w_y^i	(0.4,0.2,0.2,0.2)	(0.3,0.3,0.3,0.1)	(0.3,0.3,0.2,0.2)	

Calculate tabular data weighted averages (2) bearing in mind the constraints (9). For example,

$$W_{w_x^6}(X^1) = W_{(0.7,0.2,0.1)}(0.3,0.5,0.6) = 0.7 \cdot 0.5 + 0.2 \cdot 0.3 + 0.1 \cdot 0.6 = 0.47,$$

$$W_{w_y^{11}}(Y^5) = W_{(0.6,0.2,0.1,0.1)}(0.3,0.8,0.6,0.4) = 0.6 \cdot 0.6 + 0.2 \cdot 0.4 + 0.1 \cdot 0.8 + 0.1 \cdot 0.3 = 0.55.$$

Calculate functions W^1 and W^2 values according to the formulas (6) and (7) for each w_x and w_y pair by summing through all N objects. Write the calculated values in the Table 1 (each cell the top number is the value W^1, the bottom number – W^2).

Table 1

Values of functions W^1 (the top number) and W^2 (the bottom number) for all $14 \times 23 = 322$ weights combinations

i	w_x^1	w_x^2	w_x^3	w_x^4	w_x^5	w_x^6	w_x^7
w_y^1	2.800 0.721	2.590 0.670	2.380 0.642	2.720 0.699	2.410 0.640	2.690 0.696	2.480 0.665
w_y^2	2.640 0.676	2.410 0.618	2.200 0.584	2.540 0.644	2.230 0.578	2.510 0.637	2.300 0.601
w_y^3	2.480 0.644	2.250 0.578	2.060 0.537	2.360 0.599	2.050 0.526	2.330 0.588	2.120 0.547
w_y^4	2.360 0.643	2.050 0.558	1.780 0.494	2.040 0.512	1.590 0.459	1.870 0.477	1.640 0.461
w_y^5	2.360 0.625	2.090 0.553	1.940 0.505	2.220 0.569	1.890 0.489	2.190 0.553	1.940 0.507
w_y^6	2.260 0.621	1.930 0.529	1.660 0.456	1.860 0.473	1.490 0.413	1.690 0.430	1.520 0.409
w_y^7	2.560 0.715	2.190 0.615	1.840 0.528	1.700 0.485	1.650 0.462	1.450 0.410	1.500 0.427
w_y^8	2.280 0.621	2.030 0.544	1.820 0.491	2.080 0.554	1.790 0.469	2.050 0.534	1.800 0.483
w_y^9	2.220 0.615	1.870 0.518	1.600 0.437	1.740 0.452	1.410 0.386	1.530 0.401	1.420 0.375

i	w_x^1	w_x^2	w_x^3	w_x^4	w_x^5	w_x^6	w_x^7
w_y^{10}	2.440 0.684	2.070 0.580	1.700 0.488	1.600 0.445	1.470 0.417	1.330 0.364	1.320 0.381
w_y^{11}	2.560 0.708	2.190 0.603	1.820 0.509	1.640 0.462	1.590 0.435	1.350 0.376	1.400 0.392
w_y^{12}	2.200 0.632	2.010 0.553	1.820 0.496	2.000 0.557	1.730 0.470	1.950 0.532	1.700 0.479
w_y^{13}	2.200 0.625	1.870 0.524	1.600 0.440	1.660 0.452	1.410 0.383	1.490 0.395	1.360 0.366
w_y^{14}	2.440 0.691	2.070 0.583	1.700 0.487	1.580 0.442	1.450 0.411	1.290 0.354	1.360 0.368
w_y^{15}	2.560 0.714	2.190 0.606	1.820 0.509	1.620 0.458	1.570 0.429	1.330 0.366	1.400 0.379
w_y^{16}	3.060 0.837	2.670 0.733	2.380 0.638	1.980 0.553	2.090 0.558	1.650 0.456	1.800 0.499
w_y^{17}	2.520 0.710	2.150 0.603	1.780 0.506	1.600 0.460	1.530 0.428	1.290 0.370	1.480 0.381
w_y^{18}	2.520 0.710	2.150 0.603	1.780 0.506	1.600 0.460	1.530 0.428	1.290 0.370	1.480 0.381
w_y^{19}	3.040 0.828	2.650 0.723	2.360 0.628	1.900 0.544	2.070 0.547	1.570 0.447	1.780 0.489
w_y^{20}	3.220 0.852	2.810 0.747	2.480 0.650	2.040 0.565	2.190 0.568	1.690 0.466	1.900 0.506
w_y^{21}	4.040 1.035	3.670 0.935	3.300 0.844	2.840 0.741	2.930 0.764	2.470 0.648	2.680 0.700
w_y^{22}	4.020 1.026	3.650 0.926	3.280 0.835	2.820 0.733	2.910 0.756	2.450 0.640	2.600 0.692
w_y^{23}	4.200 1.055	3.830 0.955	3.460 0.863	3.000 0.761	3.090 0.782	2.630 0.667	2.780 0.717

i	w_x^8	w_x^9	w_x^{10}	w_x^{11}	w_x^{12}	w_x^{13}	w_x^{14}
w_y^1	2.760 0.717	3.120 0.803	2.550 0.712	2.910 0.761	3.330 0.841	3.580 0.897	4.000 0.995
w_y^2	2.600 0.657	2.960 0.748	2.390 0.650	2.750 0.701	3.170 0.786	3.380 0.843	3.840 0.945
w_y^3	2.440 0.606	2.800 0.702	2.230 0.597	2.590 0.650	3.010 0.739	3.220 0.797	3.760 0.903
w_y^4	1.820 0.477	2.120 0.547	1.790 0.497	1.890 0.511	2.230 0.576	2.420 0.631	2.860 0.725
w_y^5	2.280 0.568	2.640 0.668	2.090 0.552	2.430 0.611	2.890 0.703	3.140 0.761	3.680 0.869

i	w_x^8	w_x^9	w_x^{10}	w_x^{11}	w_x^{12}	w_x^{13}	w_x^{14}
w_y^6	1.640 0.424	1.940 0.499	1.670 0.445	1.710 0.457	2.070 0.527	2.220 0.583	2.760 0.681
w_y^7	1.260 0.369	1.480 0.383	1.650 0.430	1.370 0.371	1.430 0.383	1.540 0.426	1.900 0.497
w_y^8	2.120 0.545	2.560 0.646	1.970 0.530	2.290 0.586	2.810 0.680	3.060 0.736	3.600 0.846
w_y^9	1.500 0.389	1.820 0.468	1.550 0.409	1.570 0.419	1.950 0.492	2.140 0.547	2.680 0.649
w_y^{10}	1.140 0.319	1.360 0.341	1.430 0.387	1.170 0.324	1.250 0.343	1.380 0.392	1.820 0.472
w_y^{11}	1.140 0.323	1.300 0.336	1.470 0.390	1.170 0.319	1.190 0.330	1.340 0.372	1.720 0.449
w_y^{12}	2.040 0.539	2.520 0.639	1.850 0.522	2.210 0.576	2.730 0.670	2.980 0.723	3.520 0.834
w_y^{13}	1.400 0.377	1.780 0.455	1.430 0.395	1.470 0.403	1.870 0.475	2.060 0.528	2.600 0.631
w_y^{14}	1.080 0.300	1.220 0.319	1.370 0.368	1.090 0.297	1.170 0.314	1.300 0.361	1.720 0.443
w_y^{15}	1.060 0.304	1.160 0.314	1.390 0.371	1.090 0.292	1.070 0.299	1.200 0.339	1.600 0.419
w_y^{16}	1.360 0.381	1.080 0.313	1.630 0.471	1.250 0.341	0.950 0.260	1.020 0.267	1.040 0.280
w_y^{17}	1.100 0.311	1.240 0.326	1.490 0.375	1.110 0.302	1.210 0.315	1.260 0.355	1.660 0.436
w_y^{18}	1.080 0.316	1.160 0.322	1.510 0.378	1.090 0.296	1.090 0.300	1.160 0.333	1.540 0.411
w_y^{19}	1.300 0.372	1.020 0.308	1.630 0.462	1.190 0.333	0.930 0.257	0.960 0.267	1.040 0.286
w_y^{20}	1.400 0.387	1.080 0.316	1.650 0.473	1.230 0.340	**0.890** 0.255	0.920 **0.253**	0.960 0.263
w_y^{21}	2.180 0.570	1.680 0.465	2.470 0.656	1.950 0.513	1.430 0.392	1.260 0.354	0.960 0.266
w_y^{22}	2.100 0.562	1.620 0.459	2.430 0.648	1.850 0.507	1.350 0.387	1.300 0.351	0.980 0.269
w_y^{23}	2.280 0.588	1.800 0.484	2.530 0.670	2.030 0.529	1.530 0.409	1.340 0.367	1.040 0.279

So, we get solutions of two optimization problems (8):

$$\min W^1 = 0.890 : W_x(X) = 0.5 \cdot x_2 + 0.3 \cdot x_1 + 0.2 \cdot x_3,$$
$$W_y(Y) = 0.4 \cdot y_3 + 0.3 \cdot y_4 + 0.2 \cdot y_2 + 0.1 \cdot y_1. \quad (12)$$

$$\min W^2 = 0.253 : W_x(X) = 0.4 \cdot x_2 + 0.4 \cdot x_1 + 0.2 \cdot x_3,$$
$$W_y(Y) = 0.4 \cdot y_3 + 0.3 \cdot y_4 + 0.2 \cdot y_2 + 0.1 \cdot y_1. \quad (13)$$

Designed criteria (12) and (13) will be compared applying the sets A and B (5). We'll take different parameter $\alpha = \alpha_x = \alpha_y$ values and look for elements of the sets A and B. In the first row of each cell of the Table 2 there are sets obtained according to the formula (12), in the second row – according to the formula (13).

Table 2

Sets A and B obtained with different values α and functions W^1 (the top set), W^2 (the bottom set)

α	A	nA	B	nB
0.40	{ 2,3,4,5,6,7,9,13,14,18,19,20 } { 2,3,5,6,7,9,13,14,15,18,19,20 }	12 12	{ 1,15,16,17 } { 1,4,11,12,16,17 }	4 6
0.45	{ 2,3,9,14,19,20 } { 2,3,9,14,19 }	6 5	{ 1,5,6,7,15,17,18 } { 5,6,7,15,16,17,18,20 }	7 8
0.50	{ 2,3,9,14,19 } { 2,3,9,14,19 }	5 5	$\varnothing$ { 7,18 }	0 2
0.55	{ 2,9,14,19 } { 2,9,14,19 }	4 4	$\varnothing$ $\varnothing$	0 0
0.60	{ 9 } { 9 }	1 1	{ 2,19 } { 2,19 }	2 2

We see that both criteria (12) and (13) determine the same "best" objects { 2,3,9,14,19 } belonging to the set A. However, lower values nB (the number of "doubtful" objects) can be interpreted in favour of the criterion W^1, i. e. (12).

IV. Voting theory methods

One of the most important assumptions for the weights balancing method is determination of criteria priority (1). Usually, in practice criteria preferences are determined from expert opinions. When expert opinions are different, preferences can be determined by various methods.

In Krylovas *et al.* (2014) authors presented and in Kosareva *et al.* (2016) and Krylovas *et al.* (2016) expanded the KEmeny Median Indicator Rank Accordance (KEMIRA) method, where criteria priority set by constructing Kemeny median (see Kemeny (1959)). In the current article the priority preferences are established by different voting theory methods and they are compared with the median methods.

First, show how the winner and an outsider are determined by voting theory methods, dealing with such layouts (priorities) of 5 candidates determined by 10 voters:

$$
\begin{aligned}
&x_3 \succ x_1 \succ x_2 \succ x_5 \succ x_4; \;\; x_4 \succ x_3 \succ x_2 \succ x_1 \succ x_5;\\
&x_5 \succ x_4 \succ x_3 \succ x_2 \succ x_1; \;\; x_4 \succ x_5 \succ x_3 \succ x_2 \succ x_1;\\
&x_3 \succ x_4 \succ x_5 \succ x_2 \succ x_1; \;\; x_4 \succ x_1 \succ x_5 \succ x_3 \succ x_2;\\
&x_5 \succ x_4 \succ x_3 \succ x_2 \succ x_1; \;\; x_3 \succ x_4 \succ x_1 \succ x_2 \succ x_5;\\
&x_5 \succ x_3 \succ x_1 \succ x_4 \succ x_2; \;\; x_3 \succ x_1 \succ x_4 \succ x_5 \succ x_2;
\end{aligned}
\tag{14}
$$

It is convenient to assign each of the 5 candidates rank 1 for the last place, 2 – for the penultimate and so on. Write down the grades, as this matrix columns and in the last column write down the total amount of received grades:

$$
\begin{array}{l|cccccccccc|l}
x_1 & 4 & 2 & 1 & 1 & 1 & 4 & 1 & 3 & 3 & 4 & 24\\
x_2 & 3 & 3 & 2 & 2 & 2 & 1 & 2 & 2 & 1 & 1 & 19\\
x_3 & 5 & 4 & 3 & 3 & 5 & 2 & 3 & 5 & 4 & 5 & 39\\
x_4 & 1 & 5 & 4 & 5 & 4 & 5 & 4 & 4 & 2 & 3 & 37\\
x_5 & 2 & 1 & 5 & 4 & 3 & 3 & 5 & 1 & 5 & 2 & 31
\end{array}
\tag{15}
$$

A simple and often used method – declare the winner (the leader), the candidate having the highest score and the defeated (outsider) – the candidate having the least score. In the literature, this method is sometimes called the Borda method (see Borda, 1784). So in the investigating example the winner with respect to the Borda method is third candidate, and the defeated – the second candidate. Create another table, where type the number of places taken by each candidate:

Candidate	1 place	2 place	3 place	4 place	5 place
1 cand.	0	3	2	1	4
2 cand.	0	0	2	5	3
3 cand.	4	2	3	1	0
4 cand.	3	4	1	1	1
5 cand.	3	1	2	2	2

Having this information, a leader and outsider can be provided even easier – by the number of the first places (majority method) and last places (minority method). So the winner by the majority method will be the third candidate, but an outsider is not the only one (first or second candidate). Minority method finds the same leader – the third candidate, and an outsider – the first one.

We see that different methods differently set leaders and outsiders and sometimes do not allow to identify them unambiguously. In voting theory is well known Condorcet principle – compare each candidate to each (see Condorcet, 1785). Count the number of times the first candidate had an advantage (won duels) against the other:

$x_1 \succ x_2$	$x_1 \succ x_3$	$x_1 \succ x_4$	$x_1 \succ x_5$
5	1	3	5

The first candidate is not stronger in Condorcet principle sense than any other candidate. Similarly calculate the second and third candidates duels achievements:

$x_2 \succ x_1$	$x_2 \succ x_3$	$x_2 \succ x_4$	$x_2 \succ x_5$
5	0	1	3
$x_3 \succ x_1$	$x_3 \succ x_2$	$x_3 \succ x_4$	$x_3 \succ x_5$
9	10	5	5

The third candidate won against the first and the second by Condorcet method. He receives 2 points (the number of duels won). Calculate how many duels won the fourth and fifth candidates:

$x_4 \succ x_1$	$x_4 \succ x_2$	$x_4 \succ x_3$	$x_4 \succ x_5$
7	9	5	6
$x_5 \succ x_1$	$x_5 \succ x_2$	$x_5 \succ x_3$	$x_5 \succ x_4$
5	7	5	4

The fourth candidate gets 3 points, and the fifth – one point. The winner in this case is the fourth candidate, and outsiders are two – first and second candidates. Notice that the "best" method to set leader or outsider does not exist. This is the fundamental voting theory result known as Arrow theorem for which author in 1972 granted the Nobel Prize in economics.

V. Candidates sorting algorithms

Our goal is to provide candidates with places (ranks), depending on priorities set by voters (14). Consider 6 sorting algorithms based on Condorcet, Borda, the majority and minority methods presented in Table 3. In all cases, the algorithm terminates if the leader or the outsider are determined ambiguously. In addition, we examine two more sorting algorithms on the basis of Kemeny median. To each priority $X^{(r)}: x_{i1}^{(r)} \succ x_{i2}^{(r)} \succ ... \succ x_{in}^{(r)}$ corresponds the square matrix:

$$A^{(r)} = \left(a_{ij}^{(r)}\right)_{n\times n},\ a_{ij}^{(r)} = \begin{cases} 1, x_i^{(r)} \succ x_j^{(r)}, \\ 0, x_i^{(r)} \preceq x_j^{(r)}. \end{cases}$$

For example, the priority

$$x_5 \succ x_4 \succ x_3 \succ x_2 \succ x_1 \text{ corresponds matrix } \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{pmatrix}.$$

Kemeny distance between priorities $X^{(r1)}$ and $X^{(r2)}$ is defined as:

$$\rho_K\left(X^{(r1)}, X^{(r2)}\right) = \sum_{i=1}^{n}\sum_{j=1}^{n}\left|a_{ij}^{(r1)} - a_{ij}^{(r2)}\right|. \tag{16}$$

Kemeny median of the priorities set $\left\{X^{(1)}, X^{(2)}, ..., X^{(r)}\right\}$ is called the priority M_K for which the sum $\sum_{i=1}^{r}\rho_K\left(X^{(i)}, M_K\right)$ gains it's minimum value between all possible priorities. So, when constructing a median of priorities set, we have one more candidate sorting algorithm denoted by K.

Consider the other distance between the different priorities $X^{(r1)}$ and $X^{(r2)}$. Treat the relevant permutations as vectors:

$$\rho_P\left(X^{(r1)}, X^{(r2)}\right) = \sum_{i=1}^{n}\left|x_i^{(r1)} - x_i^{(r2)}\right|. \tag{17}$$

For example, $\rho_P\left(x_1 \succ x_2 \succ x_3, x_2 \succ x_3 \succ x_1\right) = |1-2| + |2-3| + |3-1| = 4.$ Similarly, define the median M_P and we have the additional sorting algorithm denoted by P. As in the case of Kemeny median this *permutations median* can be not the only one. It will be treated that the algorithm does not perform sorting, since it terminates.

Table 3

Description of 8 sorting algorithms

Number	Method	Algorithm
1	$C\uparrow$	Condorcet method determines the *outsider* and it is removed from the list of candidates. Again *outsider* determined of the remaining candidates, and so on.
2	$C\downarrow$	Condorcet method determines the *leader* and it is removed from the list of candidates. Again *leader* determined of the remaining candidates, and so on.
3	D	Majority method determines the *leader* and it is removed from the list of candidates. Again *leader* determined of the remaining candidates, and so on.
4	M	Minority method determines the *outsider* and it is removed from the list of candidates. Again *outsider* determined of the remaining candidates, and so on.

Number	Method	Algorithm
5	$B\uparrow$	Borda method determines the *outsider* and it is removed from the list of candidates. Again *outsider* determined of the remaining candidates, and so on.
6	$B\downarrow$	Borda method determines the *leader* and it is removed from the list of candidates. Again *leader* determined of the remaining candidates, and so on.
7	K	Kemeny median based method when distance function determined by formula (16).
8	P	Permutations median based method when distance function determined by formula (17).

VI. The conditions and results of the experiments

In this article, a statistical test to compare 8 provided priority setting algorithms was conducted using the Monte Carlo method. We considered the cases where r experts (voters) are ranking 3, 4 or 5 candidates and sorting (voting) results are processed by each algorithm $C\uparrow, C\downarrow, D, M, B\uparrow, B\downarrow, K, P$. The number of experts varied from 3 to 10. The set of priorities (voting result) is generated as follows. Each expert with a certain probability p ($p = 0.3 - 0.8$) can select only one fixed combination, which we treat as a "right" or, with a probability of $1-p$ any of "wrong" combinations. The numbers of "wrong" combinations are 5, 23 or 119 depending on the number of candidates (3, 4, 5). Suppose that all "wrong" combinations probabilities are equal. Each algorithm may lead to T – sorting result coincided with the "true" combination, F – false, the result does not match the "true" combination, N – sorting algorithm failed.

Tables 4-5 present the results of experiments in which every vote was randomly generated 1000 times and values of T, F, N are written in the columns in the appropriate cell of the table. Another computer voting result is written in another column cell. Every group of 1000 voting experiments was repeated by 10 times.

Table 4

Voting results of 8 sorting algorithms for 3 candidates, 3 experts, $p = 0.5$

1	$C\uparrow$	T F N	569 399 32	597 362 41	588 373 39	573 394 33	572 395 33	589 376 35	578 384 38	591 370 39	577 380 43	600 364 36
2	$C\downarrow$	T F N	569 399 32	597 362 41	588 373 39	573 394 33	572 395 33	589 376 35	578 384 38	591 370 39	577 380 43	600 364 36
3	D	T F N	546 315 139	569 304 127	564 291 145	553 309 138	544 319 137	558 307 135	544 294 162	560 287 153	539 309 152	570 281 149
4	M	T F N	536 308 156	572 288 140	555 289 156	534 318 148	538 313 149	559 288 153	553 308 139	556 292 152	551 297 152	576 301 123

5	$B\uparrow$	T	487	524	519	505	506	514	509	523	505	535
		F	360	324	354	375	357	353	352	338	349	332
		N	153	152	127	120	137	133	139	139	146	133
6	$B\downarrow$	T	493	524	514	522	508	515	512	516	520	518
		F	367	336	348	362	363	341	360	338	351	338
		N	140	140	138	116	129	144	128	146	129	144
7	K	T	569	597	588	573	572	589	578	591	577	600
		F	399	362	373	394	395	376	384	370	380	364
		N	32	41	39	33	33	35	38	39	43	36
8	P	T	513	544	531	514	510	528	519	525	513	546
		F	224	230	207	233	237	219	218	209	226	218
		N	263	226	262	253	253	253	263	266	261	236

Table 5

Voting results of 8 sorting algorithms for 5 candidates, 10 experts, $p = 0.5$

1	$C\uparrow$	T	749	762	735	731	738	749	786	757	769	758
		F	27	34	27	32	36	40	21	31	34	33
		N	224	204	238	237	226	211	193	212	197	209
2	$C\downarrow$	T	677	668	642	657	659	666	686	665	681	665
		F	12	13	11	16	9	8	4	8	10	13
		N	311	319	347	327	332	326	310	327	309	322
3	D	T	829	830	825	812	818	833	848	836	837	828
		F	32	26	25	35	37	28	34	26	27	36
		N	139	144	150	153	145	139	118	138	136	136
4	M	T	842	843	823	828	814	828	865	833	833	832
		F	20	31	28	29	42	29	22	30	34	20
		N	138	126	149	143	144	143	113	137	133	148
5	$B\uparrow$	T	579	596	557	562	559	571	601	574	600	543
		F	190	180	195	187	195	176	182	200	162	203
		N	231	224	248	251	246	253	217	226	238	254
6	$B\downarrow$	T	577	565	578	549	568	579	609	588	600	578
		F	183	176	192	187	176	189	163	186	178	192
		N	240	259	230	264	256	232	228	226	222	230
7	K	T	756	762	739	738	742	753	788	757	772	760
		F	30	36	30	35	37	42	23	39	37	36
		N	214	202	231	227	221	205	189	204	191	204
8	P	T	874	883	866	862	857	881	888	871	879	869
		F	46	38	42	54	53	42	45	40	48	50
		N	80	79	92	84	90	77	67	89	73	81

For the statistical analysis 100 experiments were performed with various values of experts number, probability of the correct decision and the number of candidates. Then average values of the numbers of correct decision, wrong decision and failed voting procedures were calculated for 8 sorting algorithms.

In Figure 1 the average numbers of correct decisions subject to number of experts is presented in the case of 5 candidates, 3, 5, 7, 10 experts and $p = 0.6$. In the Figure 2 we see the average numbers of failed voting procedures depending on the number of experts for the same experiment. With the growing number of experts P (8) method gives the highest percentage of correct decisions. In addition, the growing number of experts in all methods lead to the increasing percent of correct solutions. The number of failed voting procedures decreases with the growing number of experts, except the both Condorcet (1), (2) and K (7) methods for 10 experts. The best results demonstrate P (8) and K (7) methods. In Figure 3 the dependence of the average number of correct decisions on the probability p value is depicted for 4 candidates, 5 experts and $p = 0.4, 0.5, 0.6, 0.7, 0.8$ and in Figure 4 – the average number of failed voting procedures depending on the probability p value. As the p value (expert qualification) increases, for all methods probability of obtaining the correct decision also increases. The best method is Kemeny (7) – it has the highest number of correct decisions and the lowest number of terminated procedures. The minimum number of erroneous decisions have the majority (3) and minority (4) approaches. The number of erroneous decisions can be calculated by subtracting from 100 the average number of correct decisions and the average number of failed voting procedures. Both Condorcet methods (1), (2) give identical results. The dependence of the number of correct decisions and terminated procedures on the number of candidates presented in Figures 5 - 6. With the growing number of candidates right decisions percent decline for all methods. For three candidates the largest number of correct decision derived by Kemeny (7) and both Condorcet methods, in other cases – by Kemeny (7) method. The average number of failed voting procedures grows when the number of candidates increases for all methods, Kemeny (7) method shows the lowest percentage of the terminated procedures. The minimum number of erroneous decisions in all cases gives the permutations (8) method.

VII. Conclusions

The article shows how certain MCDM problems could be solved by KEMIRA method, which allows to formulate optimization task. When there is small number of evaluating criteria, the problem can be solved by options re-selection as shown in the article when dealing with the simple example. The cornerstone moment of KEMIRA method is procedure of sorting criteria according to their importance by applying Kemeny median. Such sorting can be also performed in other ways. In this article KEMIRA type methods have been constructed using the principles of priority voting and they were compared with each other. The results of Monte Carlo statistical experiment show that KEMIRA type methods (7) and (8) are in most cases superior than voting theory methods. They give higher average number of correct decisions and lower number of failed voting procedures.

Figure 1

The average numbers of correct decisions subject to experts, 5 candidates, 3, 5, 7, 10 experts, $p = 0.6$

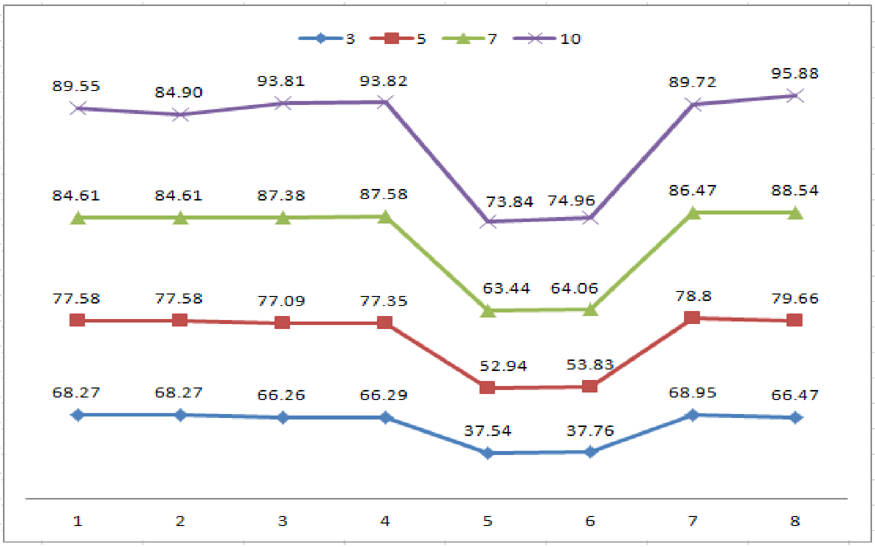


Figure 2

The average numbers of of failed voting procedures subject to experts, 5 candidates, 3, 5, 7, 10 experts, $p = 0.6$

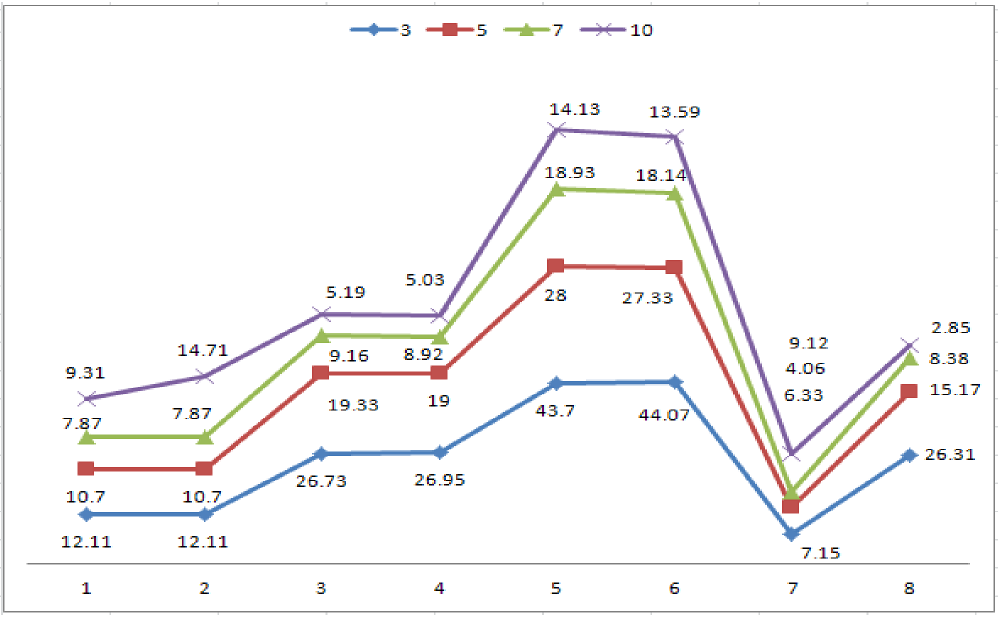


Figure 3

The average numbers of correct decisions subject to probability $p = 0.4, 0.5, 0.6, 0.7, 0.8$, 4 candidates, 5 experts

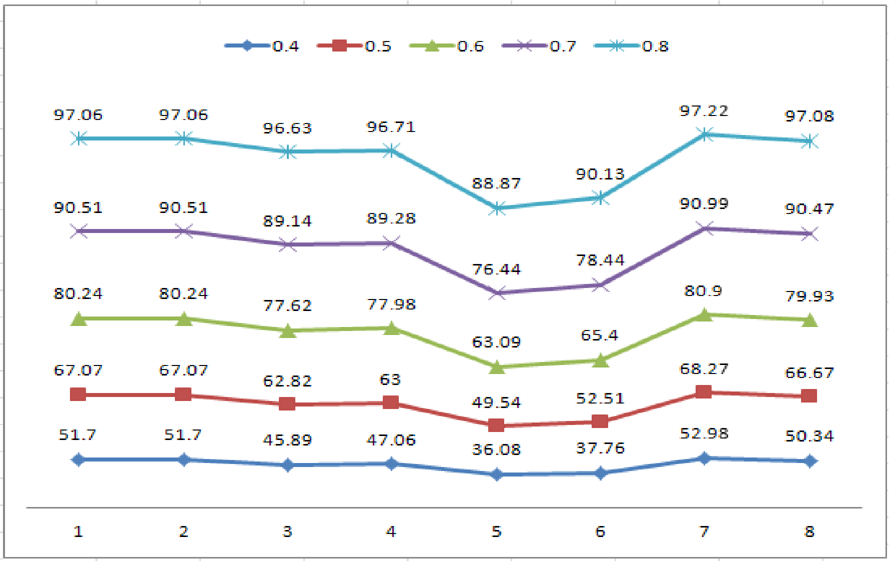


Figure 4

The average numbers of of failed voting procedures subject to probability $p = 0.4, 0.5, 0.6, 0.7, 0.8$, 4 candidates, 5 experts

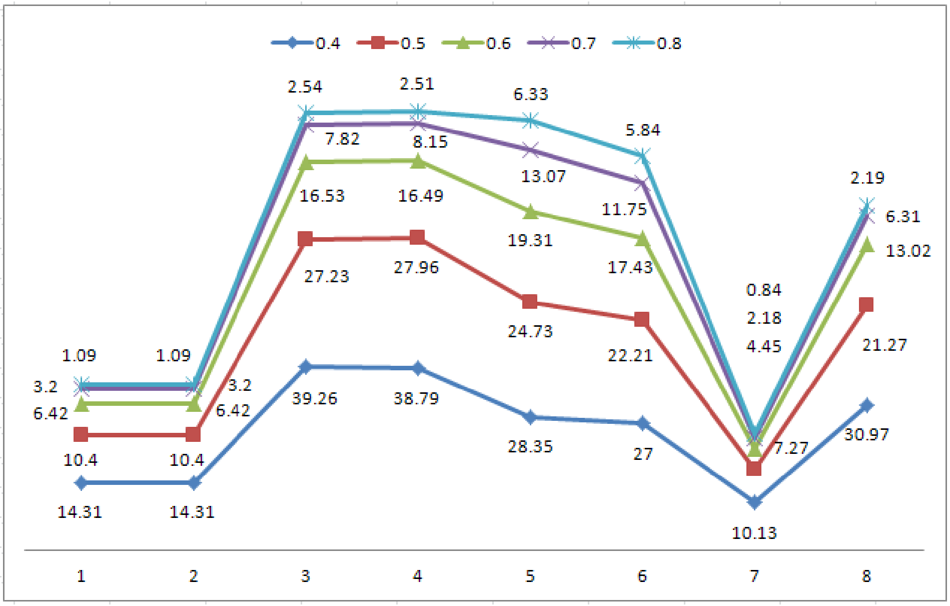


Figure 5

The average numbers of correct decisions subject to candidates, 3 experts, 3, 4, 5 candidates, $p = 0.7$

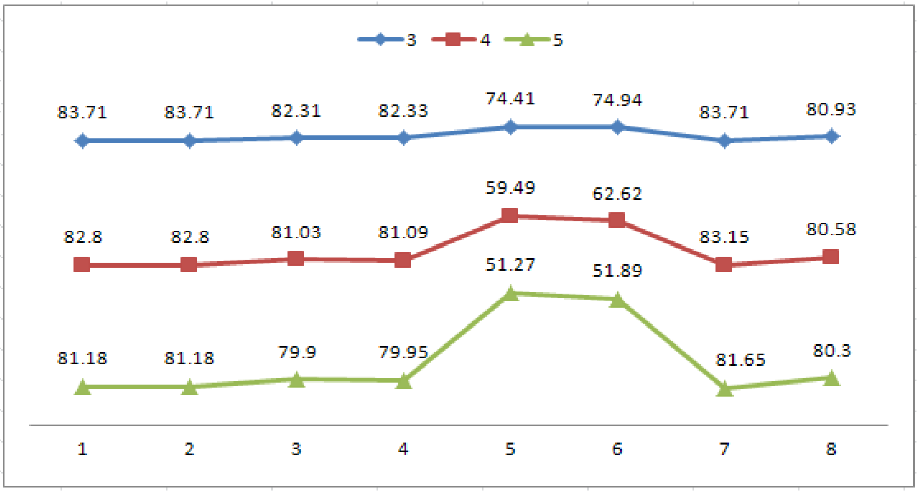
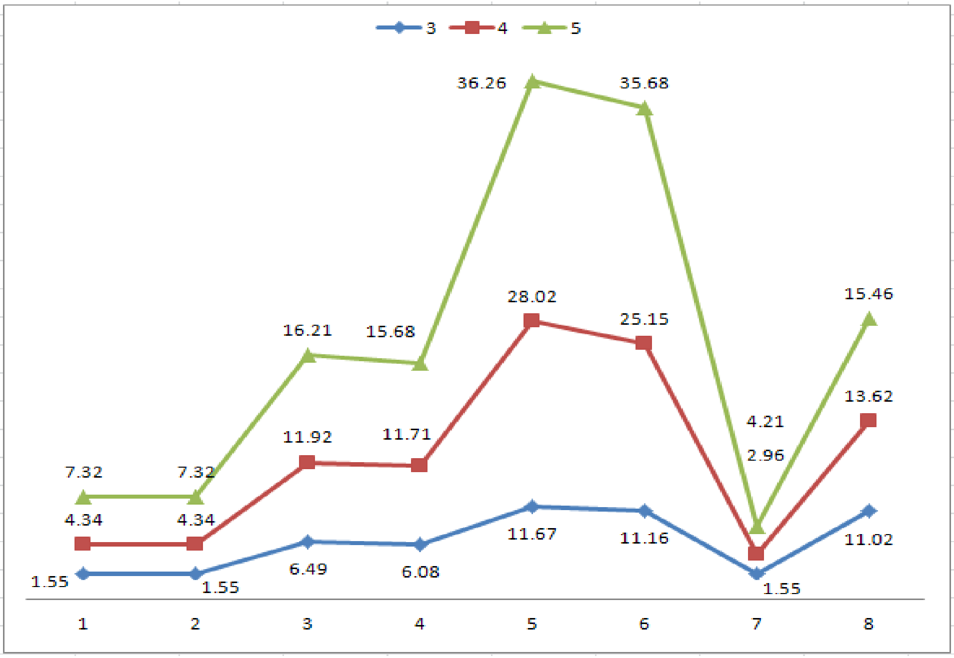


Figure 6

The average numbers of of failed voting procedures subject to candidates, 3 experts, 3, 4, 5 candidates, $p = 0.7$



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