1. NATURAL INTEREST RATE FOR THE ROMANIAN ECONOMY

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Abstract

We used a small state space model for obtaining estimates of the potential output, growth rate of the potential output and the natural interest rate. Our paper follows Laubach and Williams (2003) seminal research on natural interest rate.

Since the low interest rate environment has become a reality, are we stuck in a secular stagnation world or is just a phase of the financial cycle? We have estimated the dynamics of the Natural Interest rate (NIR) for the Romanian economy between 2004 and 2016. We have found out that the official monetary policy rate was mostly close to the natural rate of interest.

The results show that until 2010 the NIR was lower than the monetary policy rate explaining why the output grew faster than its potential value. When the real interest rate is below its equilibrium value, there are upward pressures on inflation. We have estimated the equilibrium interest rate at 3.8%, with two percentages higher than the official monetary policy rate (1.75%). Our estimate of the equilibrium interest rate after 2010 was higher than the official policy rate. In this way, some inflationary pressures may be explained. The results may also suggest that the Central Bank should have raised faster the interest rate.

In addition to the inflationary pressure, we also showed that the steady decline of the NIR after the financial crisis of 2009 coincided with an increase in the trend growth of the potential output. Since NIR is unobservable, the uncertainty around the natural rate is large and our results confirm similar findings from the economic literature.

Keywords: natural rate of interest, Kalman filter, state space model, unobserved components

JEL Classification: E32, C32

1. Introduction

The idea of a “natural rate of interest” (NIR) or the “equilibrium real interest rate” (ERR) was firstly suggested by Wicksell (1936) as the real short-term interest rate consistent with output at a stable inflation. NIR is the interest rate which causes neither overheating nor recession. Although the natural rate of interest is unobservable, its estimates are useful for setting the

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monetary policy rate, since Central Banks have to measure the level of economic variables in relation to their equilibrium values for assessing the pressures of inflation relative to its target. If output grows faster than its potential value, then the real interest rate will be below its natural value and there will be an upward pressure on inflation. In this case, the Central Bank should most likely tighten the monetary policy to reduce inflation back to its target.

The idea of NIR was formalized by Taylor (1993) for devising monetary policy rules. The Taylor rules most used in practice assume a constant NIR. In the context of actual European monetary environment known as NIRP (Negative Interest Rate Policy), a Taylor rule is out of the question because of the lower bound. Due to the economic crisis of 2007-2009, the interest rates fell below the inflation rate causing negative real interest rates. In a prolonged period of lower economic growth (Summers, 2014) the NIR will turn negative. Negative output gaps and falls in commodity prices have also contributed to the low inflation in the recent years.

Time varying variables, such as the shifts in aggregate demand and supply, supply shocks (changes in energy prices or in terms of trade), trend growth of income, fiscal policy, technological change, productivity growth, demographics, household preferences, long-run global interest rates have an influence on natural interest rate. Some variables are unobservable, while others may be estimated with a significant lag, which renders their estimations fraught with errors for taking policy decisions. A low demand for capital coupled with a higher propensity to save may induce a lower output trend growth, which in turn may decrease the natural rate of interest. This is in short the “secular stagnation” view.

What are the drivers that account for the lowering of interest rates in the last decade? The structural factors accounting for the secular stagnation are: 1) the supply schedule for loanable funds (global savings), 2) demand schedule for loanable funds (global investment), 3) relative demands for save versus risky assets. Dăianu (2017) showed that due to the dynamics of structural factors and the slowdown of global economy, the real interest rates have trended lower long before the 2007-2009 crisis.

Hamilton *et al.* (2016) showed that the estimation of the NIR involves a great deal of uncertainty. They also do not support the secular stagnation view that the equilibrium rate will be around zero in the medium term.

According to the secular stagnation theory, there are structural reasons at play, such as demographic population increases coupled with decreases in the total factor productivity factor due to the rising inequality, especially after the global financial crisis of 2007-2009. Since the structural factors are bound to stay, we may expect a low interest rate environment for many years, the so-called “low for long” scenario in which low growth is keeping down the interest rate by affecting the demand for debt.

Other explanation could link the declining trend of interest rate with the pile-up of debt in the developed economies, which in turn have limited the investment growth. Furthermore, after the financial crisis due to deleveraging of the financial system, the interest rates continued to decline in line with the unconventional monetary policies pursued all around the globe. In this “financial cycle” approach we may see a rebound of the interest rate, due to the effort of the monetary authorities to restructure the economy.

We have followed Laubach and Williams methodology (2003, 2016), and since the potential output, growth rate of the potential output and the natural interest rate are unobservable, we have estimated them with a small state space model. The multivariate state space model includes a dynamic IS equation of the output gap, representing aggregate demand and an expectations-augmented Philips curve that represents the aggregate supply. The model was estimated using maximum likelihood estimation and the Kalman filter.

2. Literature review

In the context of the Taylor rules (Hofmanb *et al*. 2012; Taylor, 2014), it is hard to take into account:

- Level shifts, namely variations in the level of potential output,
- Slope changes, namely changes in the growth rate of potential output, and
- Equilibrium real interest rate is time-varying.

Accurate estimates of potential output are important in order not to make policy mistakes. Orphanides *et al*. (2002) showed that in the '70s, the US policy had been guided rather by misperceptions regarding the natural rates of interest and unemployment. The authors argued that the Fed had mismeasured the slowdown of the potential GDP growth rate around the first Oil crisis, and thus had repeatedly over-estimated the potential GDP level. Due to this undershooting error, the Fed kept the federal funds rate too low, which in turn caused persistently high inflation rate in the 1970s. Trehan and Wu (2007) showed that in the late '90s, although the inflation didn't fall when productivity accelerated, "suggests that the Fed may no longer be using rules that depend upon the level of the (unemployment) gap".

The Laubach and Williams (LW) model is a simple New Keynesian framework that jointly estimates three unobserved variables of great interest to monetary policy makers: natural interest rate (NIR), potential output and trend growth rate. LW adopt a structural methodology, which make use of the correlations among real output gap, core inflation and interest rate gap, which is the difference between real interest and its equilibrium (natural) value. The observed variables include the real GDP and core (PCEPI) inflation, while state variables include the trend growth rate, potential GDP, and a random-walk drift term mimicking households' time preferences. LW model assume an explicit relationship between the natural rate of interest and the estimated trend growth of GDP, while Orphanides *et al.* (2002) modelled the NIR with a random walk.

Some economists do not take into account the estimates of the natural rate for taking policy decisions, due to its imprecision. Laubach and Williams (2003, LW) document the great degree of uncertainty regarding estimates of the natural rate of interest. LW showed that any policy rules based on the assumption of a constant NIR or its mismeasurement lead to the imposition of wrong stabilization policies. Croitoru (2016) argues that in the case of the Romanian economy there is a high likelihood of generating biased estimates of the natural interest rate.

Neri *et al*. (2017) show the implications for the monetary policy in the medium and long run given the low interest rate environment. Holston, Laubach and Wiliams (2017) showed that NIR decreased in the advanced economies, reaching negative values for the euro area in 2016.

The results of Pescatori and Turunen (2015) indicate that the NIR turned negative well before the financial crisis of 2007-2009 and turned positive during 2014. Their projection suggests that the NIR will increase gradually, and they argue for a "low for longer" scenario.

An alternative way to estimate NIR is to use a measure of the long-term expected interest rate and subtract the expected inflation (inflation target).

The paper is structured as follows: section 3 explains the data and methodology, in section 4 we present the results and issued related to the estimation of the natural interest rate, while in section 5 we present the final remarks.

3. Data and Methodology

In order to estimate the Natural Rate of Interest, we used the following data: the core inflation was calculated as the annualized quarterly growth rate HICP excluding energy and seasonal food (source: Eurostat); GDP (chain-linked volumes, index 2010=100, source: Eurostat); imports of goods and services (chain-linked volumes, index 2010=100; source: Eurostat); Europe Brent Spot Price FOB (source: International Energy Agency); monetary policy rate (source: National Bank of Romania). The GDP is seasonally adjusted and then transformed in its natural logarithm (100*ln GDP_t).

The inflation rate of the import price was calculated as the annualized quarterly change and the imported oil price inflation rate was calculated as the annualized change in the Europe Brent Spot Price (quarterly frequency).

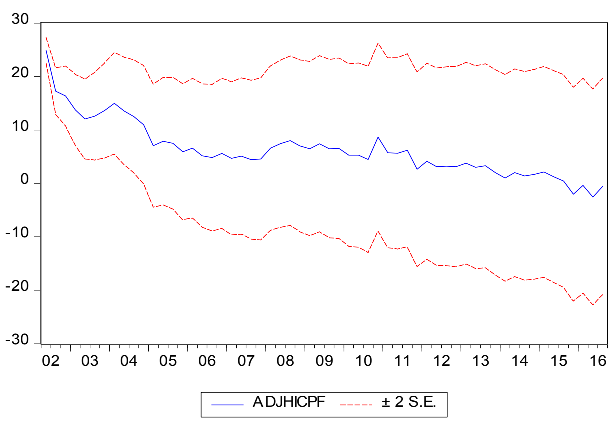
The gap of import price inflation is the difference between the import price inflation rate and the expected inflation. We did a similar calculation for the oil inflation gap. Both time series are stationary.

The monthly adjusted HICP was transformed to a quarterly rate. Then we have calculated the quarterly inflation rate. The expected quarterly inflation over the next quarter was estimated with an ARMA (1,1) model.

We have forecasted one-step-ahead inflation and the resulting values were used as expected inflation. You may see below the fit of the model for inflation. Real interest rate is the difference between the quarterly monetary policy rate and the expected quarterly inflation.

Figure 1

ARMA (1,1) model for expected Inflation



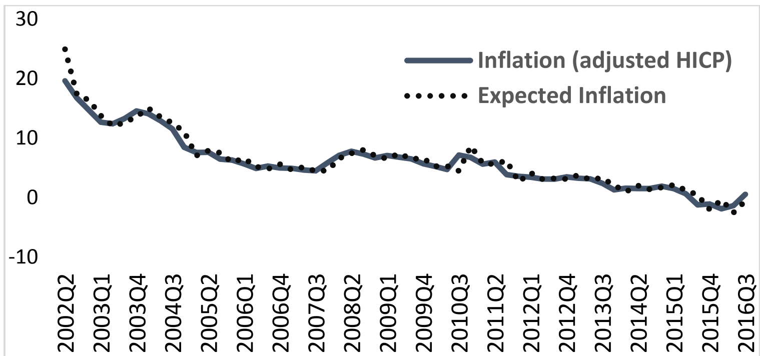
Forecast: ADJHICPF	
Actual: ADJHICP	
Forecast sample: 2002Q1 2016Q3	
Adjusted sample: 2002Q2 2016Q3	
Included observations: 58	
Root Mean Squared Error	1.177169
Mean Absolute Error	0.823714
Mean Abs. Percent Error	24.80292
Theil Inequality Coefficient	0.075811
Bias Proportion	0.073589
Variance Proportion	0.141683
Covariance Proportion	0.784729
Theil U2 Coefficient	0.972113
Symmetric MAPE	24.45317

Source: Eurostat, own calculation.

The reported statistics for the expected inflation are good; since the Theil inequality coefficient and the bias proportion are small, and the most of the bias is concentrated on the covariance proportion and Theil U2 statistics is lower than 1.

Figure 2

ARMA (1,1) model for expected Inflation



Source: Eurostat, own calculation.

The output (y_t) is decomposed into a stochastic trend component (potential output) and a stochastic cyclical variation (output gap) around the trend.

$$y_t = y_t^* + \tilde{y}_t$$

y_t^* is the potential output;
$\tilde{y}_t = y_t - y_t^*$ is the output gap;
Some researchers have also included in the IS curve equation the terms of trade for capturing the impact on the output gap. Others have modelled the output gap with an AR(1) or AR(2) process. We have opted for an AR(2) process, since we considered it to have a greater explanatory power for Romanian economy.

The IS curve

$$\tilde{y}_t = \alpha_{y1}\tilde{y}_{t-1} + \alpha_{y2}\tilde{y}_{t-2} + \frac{\alpha_r}{2}\sum_{j=1}^{2}\left(r_{t-j} - r_{t-j}^*\right) + \varepsilon_{1t}$$

where:
y_t is the log of real Romanian GDP;
$\tilde{y}_t = y_t - y_t^*$ is the output gap;
The expected real interest rate, $r_{t,}$ is defined as the difference between the monetary policy rate, (i_t), and the expected inflation rate for the next quarter (one-step ahead inflation forecast).
$r_t = i_t - E\pi_{t+1}$ is the real interest rate, where i_t is the quarterly monetary policy rate and $E\pi_{t+1}$ are the inflation expectations. The difference between $r_{t-j} - r_{t-j}^*$ is real interest rate gap. The r_t^* is the time-varying equilibrium real interest rate (NIR).
The aggregate supply side is represented by a backward Philips curve, where the inflation expectations are assumed to be driven by a backward process; hence, the inclusion of lagged inflation terms. The impact of excess demand on inflation is captured by the first lag of the output gap. We did similarly for the import prices and for oil price.

The Philips Curve

$$\pi_t = \sum_{i=1}^{8} b_{\pi i}\pi_{t-i} + b_y\tilde{y}_{t-1} + b_l(\pi_t^l - \pi_t) + b_o(\pi_{t-1}^o - \pi_{t-1}) + \varepsilon_{2t}$$

where:

π_t is the annualized quarterly change in the adjusted HICP;

π_t^l is the core import price inflation rate;

π_t^o is the crude imported oil price inflation rate;

with the restriction $\sum_{i=1}^{8} b_{\pi i} = 1$.

Orphanides *et al.* (2002) shows that the multivariate models of NIR may provide inconsistent results, due to the restriction that the sum of the coefficients on lagged inflation in the inflation equation equals unity.

The natural interest rate (equilibrium real interest rate), r_t^*, varies over time, in response to shifts to households' time preferences and the growth rate of potential output (g_t) and follows a random walk:

$r_t^* = cg_{t-1} + r_{t-1}^* + \varepsilon_{3t}$. where: g_t is the trend growth rate.

Potential output, y_t^*, evolves according to a simple law of motion:

$$y_t^* = y_{t-1}^* + g_{t-1} + \varepsilon_{4t}$$

where: g_{t-1} represents the quarterly trend growth rate from last quarter, which follows a random walk.

$$g_t = g_{t-1} + \varepsilon_{5t}$$

We assume that the errors $\varepsilon_{1t} \dots \varepsilon_{5t}$ are serially uncorrelated and not correlated with each other.

The State Space representation

Since output gap is $\tilde{y}_t = y_t - y_t^*$, we have substituted in the IS equation:

$$y_t = y_t^* + \alpha_{y1}(y_{t-1} - y_{t-1}^*) + \alpha_{y2}(y_{t-2} - y_{t-2}^*) + \frac{\alpha_r}{2}\sum_{j=1}^{2}\left(r_{t-j} - r_{t-j}^*\right) + \varepsilon_{1t}$$

The state vector is defined as

$$S_t = (y_t^*, y_{t-1}^*, y_{t-2}^*, g_t, g_{t-1}, r_t^*, r_{t-1}^*)'$$

The law of motion for the state vector is

$$\boldsymbol{S_t} = \boldsymbol{\Gamma S_{t-1}} + \boldsymbol{\Pi \eta_t}$$

$$\begin{bmatrix} y_t^* \\ y_{t-1}^* \\ y_{t-2}^* \\ g_t \\ g_{t-1} \\ r_t^* \\ r_{t-1}^* \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & c & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} y_{t-1}^* \\ y_{t-2}^* \\ y_{t-3}^* \\ g_{t-1} \\ g_{t-2} \\ r_{t-1}^* \\ r_{t-2}^* \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \varepsilon_{4t} \\ \varepsilon_{5t} \\ \varepsilon_{3t} \end{bmatrix}$$

We assume that the shocks are uncorrelated. Changes in the trend rate, (ε_{5t}), have the greatest effect on the evolution of the series, follows by the level shock, (ε_{4t}), while the measurement error, (ε_{2t}), has the lowest contribution.

The measurement equation

$$\boldsymbol{Y_t = A'X_t + H'S_t + \zeta_t}$$

where Y_t vector includes the dynamics of IS curve and Philips curve

$$\begin{bmatrix} y_t \\ \pi_t \end{bmatrix} = \begin{bmatrix} \alpha_{y1} & \alpha_{y2} & \alpha_r/2 & \alpha_r/2 & 0 & 0 & 0 & 0 & 0 \\ b_{y3} & 0 & 0 & 0 & b_{y1} & b_{y2} & 1-b_{y1}-b_{y1} & b_{y4} & b_{y5} \end{bmatrix} X_t +$$

$$+ \begin{bmatrix} 1 & -\alpha_{y1}-\alpha_{y2} & -c*\alpha_r/2 - c*\alpha_r/2 & -\alpha_r/2 & -\alpha_r/2 \\ 0 & -b_y & 0 & 0 & 0 & 0 & 0 \end{bmatrix} S_t + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix}$$

where X_t is the vector of observed variables and is

$X_t = (y_{t-1}, y_{t-2}, r_{t-1}, r_{t-2}, \pi_{t-1}, \pi_{t-2}, \pi_{t-5}, \pi^o_{t-1}, \pi^l_t)'$

4. Model Estimation and Estimation Issues

The model was estimated with the Kalman filter using ML procedure. A complex issue in the estimation is finding the values for the standard deviation of the trend growth of potential output and for the households' time preferences, (z_t), the so-called "pile-up issue" as discussed in Stock (1994). Stock-Watson (1998) proposed a median unbiased estimator to solve the issue, which was adopted by the LW.

LW overcame the "pile-up issue" by imposing two assumptions. They assumed that the standard deviation of the trend growth of potential output, (σ_5), is the standard deviation of the *i.i.d* shocks in growth rate of potential output, divided by the standard deviation in the potential GDP level and the value obtained is the standard deviations of the quarterly trend growth rate. For annualized trend growth rate, the value was multiplied by 4.

$$\frac{\sigma_5}{\sigma_4} \equiv \lambda_g$$

The standard deviation for the households' time preferences, (z_t) is denoted by λ_z and it is the

$$\frac{\sigma_3}{\sigma_1}\frac{\alpha_r}{\sqrt{2}} \equiv \lambda_z$$

Because of the „pile-up" issue, σ_z and σ_g are biased towards 0. LW applied the Kalman filter in the first step to estimate the natural rate of output, omitting the real-rate gap term in the output gap equation and assuming that the trend growth rate, g, is constant. From this preliminary estimate of the natural rate of output, they then compute the median unbiased estimate of λ_g. In the second step, LW imposed the estimated λ_g from the first step and then estimated the full model, including the real-rate gap in the output gap equation, but under the assumption that z is constant. From this estimation, LW computed the median unbiased estimate of λ_z. In the final step, LW imposed the estimated λ_g and λ_z and estimated the remaining model parameters by MLE.

For the Romanian economy, we have imposed a structural break in the real GDP series for the fourth quarter of 2008; otherwise the projected trend of GDP would have been higher than the real GDP, thus mistakenly indicating a negative output gap.

5. The Results

The estimation of the state space model converged and the values of the parameters are significant. The results are presented in Table 1.

Table 1

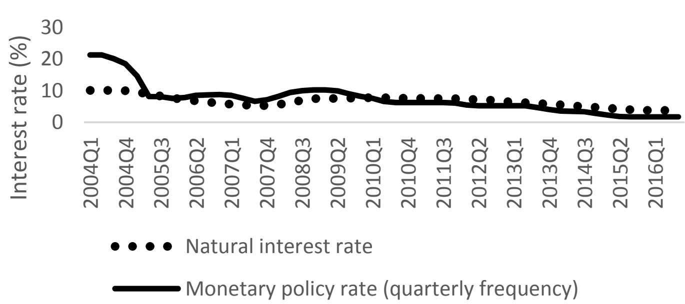
Model's parameters

Variable	Coeff	Std.Error	T-Stat	Signif
α_{y1}	1.37	0.14	9.77	0.00
α_{y2}	-0.45	0.13	-3.55	0.00
α_r	0.09	0.07	1.20	0.23
b_{y1}	0.84	0.12	7.29	0.00
b_{y2}	-0.21	0.11	-1.86	0.06
b_{y3}	0.15	0.03	4.35	0.00
b_{y4}	0.00	0.00	0.58	0.56
b_{y5}	-0.01	0.01	-1.22	0.22
c	4.00	0.00	0.00	0.00
σ_1	1.25	0.23	5.44	0.00
σ_2	0.75	0.08	9.07	0.00
σ_4	-0.42	0.59	-0.71	0.48
λ_g	0.04	0.00	0.00	0.00
λ_z	0.08	0.00	0.00	0.00

Source: Own calculation.

Figure 3

The Natural Interest Rate vs. Monetary Policy Rate



Source: Own calculation.

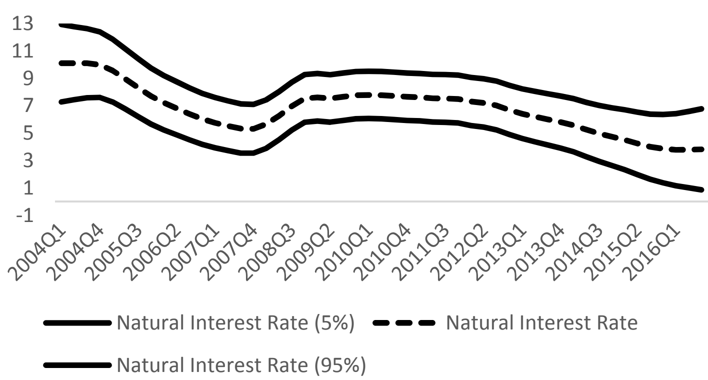
The model results show that the natural interest rate was close to the monetary policy rate. Until 2010, the NIR was lower than the monetary policy rate, showing that the output grew faster than its potential value. Since 2010, the estimate of the NIR are higher than the official policy rate. Our estimate for the NIR in the third quarter of 2016 was 3.8%, with two percentages higher than the official monetary policy rate (1.75%).

We have plotted in Figure 5 the output gap and the annual growth of the potential GDP that is around 1.65%.

The results show that there is a lot of uncertainty around Natural Interest Rate, as shawn in Figure 4. Although our estimate for NIR in 2016 is 3.8%, the uncertainty ranges between 1% and 6.7%.

Figure 4

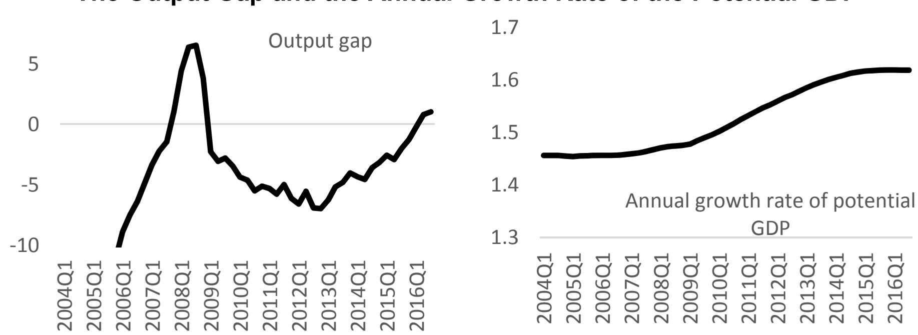
Uncertainty around the Natural Interest Rate



Source: Own calculation.

Figure 5

The Output Gap and the Annual Growth Rate of the Potential GDP

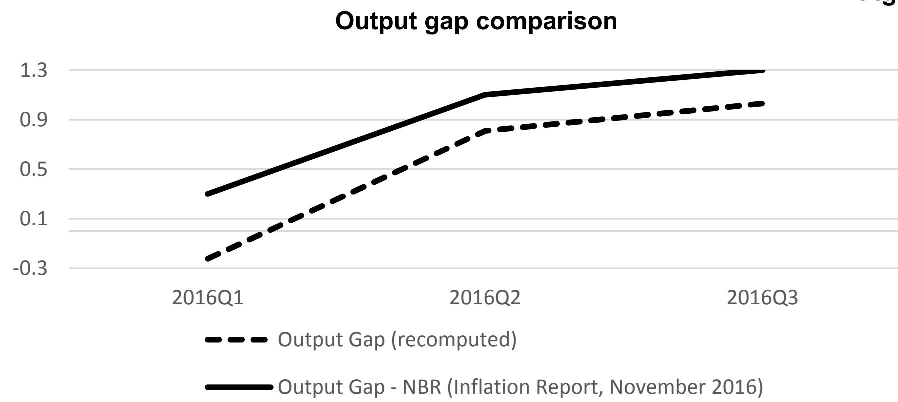


Source: Own calculation.

In Figure 5, we compared our output gap estimate with the official output gap computed by National Bank of Romania (Inflation Report, November 2016). The differences are small and show the same dynamics of the output gap, although different models calculate them.

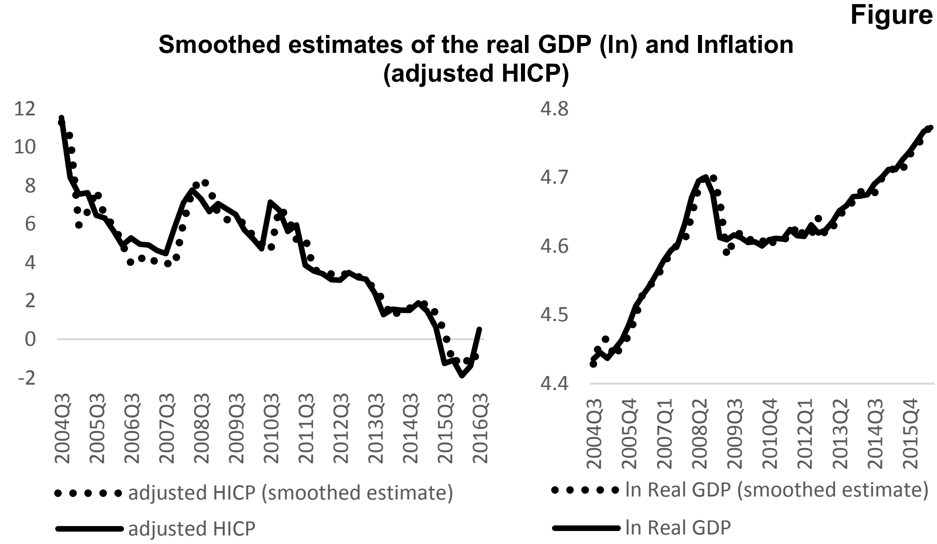
The smoothed estimates of real GDP (ln) and the inflation (adjusted HICP) show that the model yields a realistic description of the observables, while the autocorrelation of prediction errors is not significant (see Figure 7).

Figure 6



Source: Own calculation.

Figure 7

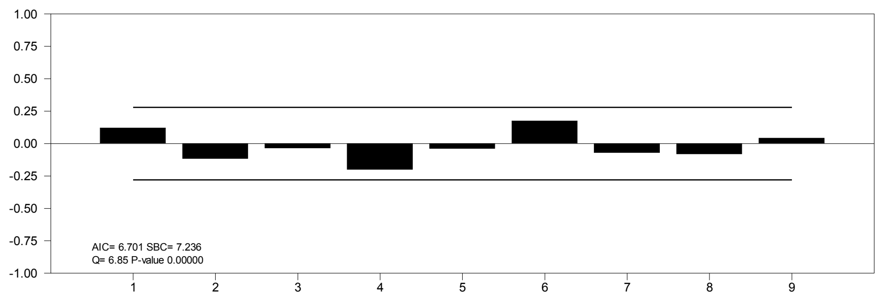


Source: Own calculation.

Since the differences between the smoothed estimates and the observed times series are very small, the prediction errors are also very small and do not exhibit serial correlation, as can be seen in Figure 7.

Figure 8

Autocorrelation of prediction errors

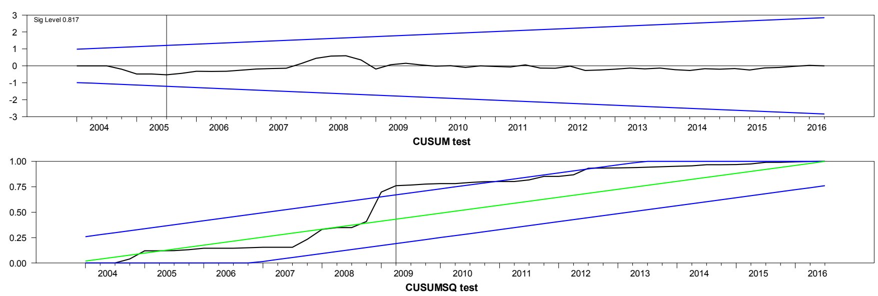


Source: Own calculation.

The CUSUM tests for the model do not show any systematic change in the variance, with the exception of the fourth quarter of 2009, which signals the recession period.

Figure 9

CUSUM TESTS



CUSUM Tests

Source: Own calculation.

6. Conclusions

The natural rate of interest is a medium-run benchmark that allows policymakers and researchers to pass a judgement about whether the actual rates are too high or too low. Since other unobservable factors influence the natural rate of interest there is a lot of uncertainty about its trajectory.

Instead of using a large-scale DSGE model for inferring the natural interest rate for Romanian economy, we have jointly estimated the potential output, the growth of the potential output and the natural rate of interest using a small state space model for Romanian economy from 2004 until third quarter of 2016. The parameter estimates were all significant with the expected sign. We have compared the actual dynamics of output and inflation with the smoothed estimates from the actual and the prediction errors do not exhibit serial correlation. The CUSUM tests also indicated that the coefficients are stable.

Although there is a great amount of uncertainty about it, the estimate of the natural rate of interest allows to show the state of the Romanian economy and provides an alternative way to comment on the monetary policy.

The steady decline of the NIR after the financial crisis of 2009 coincided with an increase in the trend growth of the economy. Besides that, our results indicate that the uncertainty around the natural rate is quite large.

The plot of the natural rate of interest clearly shows that until 2010 the natural interest rate (NIR) was lower than the monetary policy rate, thus explaining why the output grew faster than its potential values.

Since after the global financial crisis of 2007-200, the low interest rate environment has become a reality, many economists tried to develop theories and make forecasts for the long-term interest rate. Now there are two main prevailing theories: the secular stagnation view and the financial cycle one.

From this point of view, we were interested to see how the unobserved, equilibrium interest rate evolved for the Romanian economy after the financial crisis. Our estimate of the equilibrium interest rate after 2010 was higher than the official policy rate. We have estimated the equilibrium interest rate at 3.8%, with two percentages higher than the official monetary policy rate (1.75%). Since the official monetary policy rate was lower than the equilibrium interest rate, some inflationary pressures emerged.

The results suggest that the Central Bank should have raised faster the interest rate in order to stave off inflation. On the other hand, the official policy rate was constrained also by the low inflation environment pervasive to the European economy after 2010.

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