



DEBT CEILING AND EXTERNAL DEBT SUSTAINABILITY IN ROMANIA: A QUANTILE AUTOREGRESSION MODEL¹

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Abstract

In this paper we investigate the external debt sustainability using a quantile autoregression (QAR) model. QAR is a new type of econometric models used to separate periods of nonstationarity from the stationarity ones. This kind of model allows us to identify various trajectories of external debt that are compatible with indebtness sustainability. We use such trajectories to construct a debt ceiling, that is, the largest value of external debt that does not jeopardize long-run indebtness sustainability. We make out-of-sample forecast of such a ceiling and we present the debt ceiling as a “debt-warning system” which could be used by policy makers interested in keeping the external debt on a sustainable path. We illustrate the applicability of such econometric tool using Romanian data. Also, we used the R programming language for part of our statistical computing and graphics presented in this study.

Keywords: external debt, quantile autoregression, local sustainability, global sustainability, gross domestic product

JEL Classification: C21, F34

1. Introduction

Besides the stability of the Euro Area, debt sustainability is one of the most interesting topics discussed in the current macroeconomic context. This topic is sensitive for countries in South-Eastern Europe and beyond, whereas in recent years the

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economic growth of these countries was based mainly on consumption and on increase in construction volume. For our country, the rapid growth of the external debt is even a more sensitive issue because there is almost no infrastructure development.

In our opinion, it is more important to analyze the sustainability of the external debt than that of the public debt in the case of Romania, because more than a half of the private debt (46 billion euros) is actually contracted by the Ministry of Public Finance. The Ministry issued 22.5 billion euros bonds on the domestic market, which were bought by banks. Also, the local authorities borrowed 2 billion euros from banks.

External debt sustainability is measured by some ratios or indicators. For example, the ratio of total external debt to GDP is an important indicator. To achieve sustainability of debt the ratio of total external debt to GDP must be stabilized. External debt sustainability is a key element in analyzing the financial stability of a country's economy. The role of external debt sustainability increases especially when we are dealing with a country whose economy is in transition.

The objective of our study is to test for sustainability of Romania's external debt by using a method which examines global and local stationarity simultaneously. These tests are based on a model of quantile autoregression.

The rest of the study is organized as follows. The second part presents a short literature review. The third part discusses the macroeconomic and econometric methodological issues. The fourth and fifth parts present the data problems, the empirical results and the conclusions. The most important aspects dealt with are pointing out that the external debt is usually characterized by local sustainability, but not always by global sustainability⁵.

II. Literature Review

The liquidity crisis of the last decade have determined the government experts, the experts from the central banks and those from the IMF to draft new methods for assessing the liquidity and solvency risks and to analyze the sustainability of the external debt. Studies on the sustainability of external debt are the preserve of international financial organizations and central banks.

The starting point was the IMF framework on "sustainability assessment", paper approved in May 2002 by Timothy Geithner - Director of the IMF at the time, currently the U.S. Secretary of State Treasury. The last version of this framework was released in cooperation with the International Development Association in January 2010. IMF experts used sensitivity analysis to highlight the movements of macroeconomic indicators that affect debt sustainability.

Other interesting studies on debt and external debt sustainability were written by:
- Wyplosz C. (2007) – alternatives to IMF approach: Value at risk stress tests. He proposed to construct stress tests properly by taking into account the historical

⁵ *Global sustainability refers to the external debt sustainability of countries on long term, although on short term there might be periods of unsustainability. If a country's external debt is sustainable throughout the global period (decades) there might intervene short periods of time (a few quarters) during which it seems to be unsustainable.*

interdependence among these variables rather than tailoring shocks on the basis of the historical evolution of individual variables. An example is the combined shock, which involves a simultaneous deterioration in the current account, interest rate and GDP growth;

- Lima L. *et al.* (2006) – a quantile autoregression approach. They introduced the debt ceiling concept as a “debt warning system” to be used by policy makers interested in keeping the debt on a sustainable path.

- Yilanci V. and B. Özcan (2008) – a nonlinear approach, the Threshold Autoregressive Model. They have investigated the long run sustainability of Turkey's external debt by using a method suggested by Caner and Hansen (2001). They determined that if the DEX/GDP ratio shows an increase by more than 3% between the previous quarter and the second quarter before, unsustainability would occur. The result of their paper was that the external debt of Turkey was unsustainable at that moment.

- Roubini N. (2001) – a qualitative assessment of the debt sustainability. He stated that while the “practical criterion” for external and public debt sustainability (a measure of the primary gap and the trade balance gaps) provides a useful benchmark (i.e. debt is not sustainable if its ratio to GDP is growing over time without bounds), it does not directly provide a tool to assess whether a certain stock of debt is sustainable or not. As long as the debt to GDP ratio is stabilized on the medium term, it is considered as sustainable regardless of its level; i.e. a debt to GDP ratio of 150% is as sustainable as one of 50%.

The quantile autoregression model was developed by Koenker. In 1978, Koenker and Bassett published “Regression Quantiles”, which was the starting point for all regression models based on quantiles. Then, in 2006 Koenker and Xiao released a working paper that presented for the first time the QAR model methodology. The QAR approach provides a way to examine directly how the past information affects the conditional distribution of a time series. This feature of the QAR model is fundamental to the methodology proposed in this paper, since our measure of debt ceiling (\tilde{D}_t) will be nothing else than the upper conditional quantile of the external debt that satisfies the transversality condition of non-Ponzi game. Compared to the QAR approach, other non-linear methods such as the smooth transition autoregressive (STAR), threshold autoregressive (TAR) or Markov switching are not able to estimate conditional quantiles since they were originally proposed to estimate nonlinear models for conditional means (or variance).

III. Methodological Issues

The approach used to analyze sustainability consists in testing if the external debt is a stationary process. A necessary and sufficient condition for sustainability is that the discounted external debt-to-GDP ratio should be a stationary zero-mean process. External debt is considered sustainable if the country's intertemporal balance constraint is satisfied.

1. Theoretical Model

According to Uctum and Wickens (2000), a necessary and sufficient asymptotic condition for sustainability is that while n goes to infinity, the expected value of the discounted debt-to-GDP ratio converges to zero. This condition is usually known in the literature as the transversality condition (or the non-Ponzi-scheme condition), and can be summarized as:

$$\lim_{n \rightarrow \infty} E_t \frac{d_{t+n}}{(1 + \rho)^n} = 0 \tag{1}$$

where: d - foreign debt, ρ - the discount factor, E - the expectation.

2. Econometric Model

The papers elaborated in 2006 by Koenker and Xiao introduced the so-called quantile autoregression (QAR) model. This model is a random coefficient time series model whose autoregressive coefficients parameters are functionally dependent and may vary over the quantiles $\tau \in (0; 1)$. The QAR model expands the modeling options for time series that display asymmetric dynamics or local persistency.

In this paper, we show how to separate nonstationary observations from stationary ones using the QAR model. Also, we test for both global and local sustainability, with global sustainability referring to a set of quantiles and local sustainability analyzing the behavior of Romania’s external debt at a fixed quantile. The latter allows us to identify trajectories of the external debt (indebtness policies) that are not consistent with external debt sustainability in the sense that if they were allowed to persist indefinitely, they would eventually violate the intertemporal restrictions.

The Quantile Autoregression Model

Let $\{U_t\}$ be a sequence of iid standard uniform random variables, and consider the p th order autoregressive process,

$$y_t = \theta_0(U_t) + \theta_1(U_t)y_{t-1} + \dots + \theta_p(U_t)y_{t-p} \tag{2}$$

where: θ_j ’s are unknown functions $[0; 1] \rightarrow \mathbb{R}$ that I will want to estimate. I will refer to this model as the QAR (p) model.

The QAR (p) model (2) can be reformulated in a more conventional random coefficient notation as,

$$y_t = \mu_0 + \beta_{1,t}y_{t-1} + \dots + \beta_{p,t}y_{t-p} + u_t \tag{3}$$

An alternative form of the model (3) widely used in economic applications is the ADF (augmented Dickey-Fuller) representation:

$$y_t = \mu_0 + \alpha_{1,t}y_{t-1} + \sum_{j=1}^{p-1} \alpha_{j+1,t}y_{t-j} + u_t \tag{4}$$

In this model, the autoregressive coefficient $\alpha_{1,t}$ plays an important role in measuring persistency in economic and financial time series. Under regularity conditions, if $\alpha_{1,t} = 1$, y_t contains a unit root and is persistent; and if $|\alpha_{1,t}| < 1$, y_t is stationary.

Estimation

Provided that the right hand side of (2) is monotone and increases in U_t , it follows that the τ th conditional quantile function of y_t can be written as,

$$Q_{y_t(\tau)}(y_{t-1}, \dots, y_{t-p}) = \theta_0(\tau) + \theta_1(\tau) y_{t-1} + \dots + \theta_p(\tau) y_{t-p}, \quad (5)$$

In the above model, the autoregressive coefficients may be τ -dependent and thus can vary over the quantiles. If the symmetric absolute value function yields the median, we may simply tilt the absolute value to produce the other quantiles. This logic suggests solving

$$\min_{\theta \in \mathbb{R}^{p+1}} \sum_{i=1}^n \rho_{\tau}(y_t - x_t^T \theta)$$

where: the function ρ_{τ} is define as:

$$\rho_{\tau}(u) = \begin{cases} \tau u, & u \geq 0 \\ (\tau - 1)u, & u < 0 \end{cases}$$

Hypothesis Testing

a. Autoregressive Order Choice

Equation (2) gives our p th order quantile autoregression model. We now present how to choose the optimal lag length p . We follow Koenker and Machado (1999) in testing for the null hypothesis of exclusion for the p th control variable τ .

$$H_0: \theta_p(\tau) = 0, \text{ for all } \tau \in T \text{ for some index set } T \subset (0,1). \quad (6)$$

Koenker and Machado (1999) state that the null hypothesis can be tested (6) using a related version of the Likelihood process for a quantile regression with respect to several quantiles.

We want to carry out a joint test about the significance of the p th autoregressive coefficient with respect to a set of quantiles T (not only at fixed quantile). Koenker and Machado (1999) suggest the Kolmogorov-Smirnov type statistics for the joint test.

b. Global Sustainability

The concept of global sustainability states that episodes of external imbalances resulting from indebttness policies not compatible with long-run external debt sustainability must be offset by periods of political responsibility so that the intertemporal balance constraint holds in the long run. In this context, Lima L. *et al.* (2006) introduce tests for global stationarity and zero unconditional mean.

b.1. Global Stationarity

An approach to test the unit root property is to examine the unit root property over a range of quantiles $\tau \in T$, instead of focusing only on a selected quantile. Then, we may construct a Kolmogorov-Smirnov (KS) type test based on the regression quantile process for $\tau \in T$. Koenker and Xiao (2004) proposed a quantile regression-based statistics for testing the null hypothesis ($H_0: \alpha_{1,\tau} = 1$) of a unit root. Lima L. *et al.* (2006) suggest approximating the limiting distribution of QKS under the null hypotheses by using the residual based block bootstrap procedure (RBB).

b.2. Unconditional Mean Test

method for dependent data named Nonoverlapping Block Bootstrap (NBB), even if the first natural attempt it would be to ignore the existence of asymmetric dynamic and estimate a symmetric regression, which is not a valid option for this model. The key feature of this bootstrap method (NBB) is that its blocking rule is based on nonoverlapped segments of the data, making it able to simulate the weak dependence in the original series y_t .

c. Local Sustainability

To test local sustainability Lima L. *et al.* (2006) use the so-called Koenker-Xiao test for the analysis of local unit root behavior. The local unit root hypothesis in the ADF representation is:

$$H_0: \alpha_1(\tau) = 1, \text{ for selected quantile } \tau \in (0,1)$$

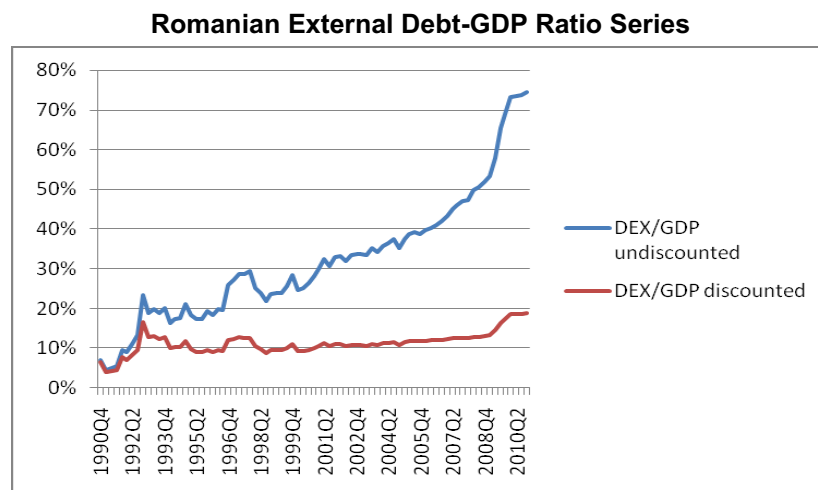
In order to test such a hypothesis, Koenker and Xiao (2004) proposed a statistic similar to the conventional Augmented Dick-Fuller (ADF) t-ratio statistic. The t_n statistics is the quantile autoregression counterpart of the ADF t-ratio test for a unit root.

IV. Empirical results

1. The Database

The methodology presented in this paper is applied to analyze the discounted Romanian external debt to GDP ratio. All data are quarterly and are obtained from the National Bank of Romania, National Institute of Statistics and Eurostat. Our sample covers the period 1990 Q4 to 2010 Q4 (81 observations). The undiscounted debt represents external debt series compiled by the Statistics Department of the National Bank of Romania, in percentage of GDP. The discounted debt is given by the undiscounted debt series multiplied by a stochastic discount factor (Figure 1).

Figure 1



The stochastic discount factor a_t , as previously mentioned in the theoretical model, is generated from ρ_t (the real interest rate adjusted for GDP growth rate), which depends on the inflation and nominal interest rates, and GDP growth rate. The inflation rate π_t is the inflation rate of the Euro Zone and the nominal interest rate i_t is Euribor for 3 months. Regarding the GDP growth rate η_t , we generate a quarterly series based on the quarterly seasonally adjustments GDP of the Euro Zone, which is released by Eurostat.

2. Results

Autoregressive Order Choice

First, we determine the autoregressive order of the QAR (p) model (5) using the Kolmogorov-Smirnov test based on LR statistics, following Koenker and Machado (1999). We start estimating the quantile regression below with $p = p_{max} = 8$. We used R language and environment for statistical computing and graphics (Table 1).

Table 1

Choice of the Autoregressive Order

Excluded variable	$\sup_{\tau \in T} L_n(\tau)$	5% critical value	10% critical value	H_0	Result
y_{t-2}	4.40	9.31	7.63	$\theta_2(\tau) = 0$	do not reject
y_{t-3}	8.76	9.31	7.63	$\theta_3(\tau) = 0$	do not reject
y_{t-4}	5.72	9.31	7.63	$\theta_4(\tau) = 0$	do not reject
y_{t-5}	10.81	9.31	7.63	$\theta_5(\tau) = 0$	reject
y_{t-6}	21.60	9.31	7.63	$\theta_6(\tau) = 0$	reject
y_{t-7}	4.78	9.31	7.63	$\theta_7(\tau) = 0$	do not reject
y_{t-8}	4.96	9.31	7.63	$\theta_8(\tau) = 0$	do not reject

Since the 7th and 8th order are not relevant, we proceed by analyzing if the sixth order covariate is relevant. We verify that the sixth autoregressive variable cannot be excluded. Thus, the optimal choice of lag length in our model is $p = 6$ and this order will be used in the subsequent estimation and hypothesis tests presented in this paper. In summary, our econometric model will be:

$$Q_{y_t}(\tau|y_{t-1}, \dots, y_{t-p}) = \theta_0(\tau) + \theta_1(\tau)y_{t-1} + \dots + \theta_6(\tau)y_{t-6} \tag{7}$$

and the associated ADF formulation is:

$$y_t = \mu_0 + \alpha_{1,t}y_{t-1} + \alpha_{2,t}\Delta y_{t-2} + \dots + \alpha_{5,t}\Delta y_{t-5} + u_t \tag{8}$$

After performing the exclusion test the result is a QAR (6) model.

Global Sustainability

The necessary and sufficient condition for the intertemporal balance constraint to hold is that the discounted external debt-GDP ratio, represented by y_t , must be a

stationary zero-mean process. If this happens, then the Romanian external debt will be globally sustainable.

a. Global Stationarity

In order to test for global stationarity, we need to test the null hypothesis ($H_0: \alpha_{1,t} = 1$) using the so-called Quantile Komogorov-Smirnoff (QKS) test. We considered 1,000 bootstrap replications.

Table 2

Results for the Global Stationarity Test

Block length	b	QKS	5% critical value	10% critical value	$H_0: \alpha_{1,t} = 1$
8		24.9508	26.1774	20.9727	reject at 10%
10		24.9508	24.2553	19.9912	reject at 5%
12		24.9508	25.1983	20.5405	reject at 10%
14		24.9508	22.2265	17.5303	reject at 5%
16		24.9508	19.9344	16.7877	reject at 5%

The results in Table 2 suggest that, at worst, the discounted Romanian external debt is globally stationary at 10% of significance, because we reject the unit root null hypothesis with significance level of 10% for all values of b.

b. Unconditional Mean Test

Now, we test the null hypothesis that the discounted debt process has zero unconditional mean (Table 3).

Table 3

Summary of Results for the Unconditional Mean Test

Block length	b	t	2.5% critical value	97.5% critical value	$H_0: \mu_y = 0$
8		39.86	30.45	107.03	do not reject at 5%
10		39.86	31.76	91.66	do not reject at 5%
12		39.86	30.26	81.68	do not reject at 5%
14		39.86	28.87	85.61	do not reject at 5%
16		39.86	32.03	83.88	do not reject at 5%

We conduct a t-test for the unconditional mean and use the NBB resampling method with 1,000 replications to compute 5% critical values. The reported results suggest that the unconditional mean of the autoregressive process is not statistically different from zero. This result associated with the QKS result for global stationarity present evidence that the external debt is globally sustainable at 10% significance and not necessarily with significance level of 5%.

Local Sustainability

To check if the Romanian external debt is locally sustainable we need a way to separate periods of stationarity from periods where y_t exhibits a nonstationary behavior. Lima L. et al. (2006) solved this problem using QAR approach. They introduced the critical conditional quantile (Table 4).

Table 4

**Results for Local Sustainability Test with Significance Levels
of 5% and 10%**

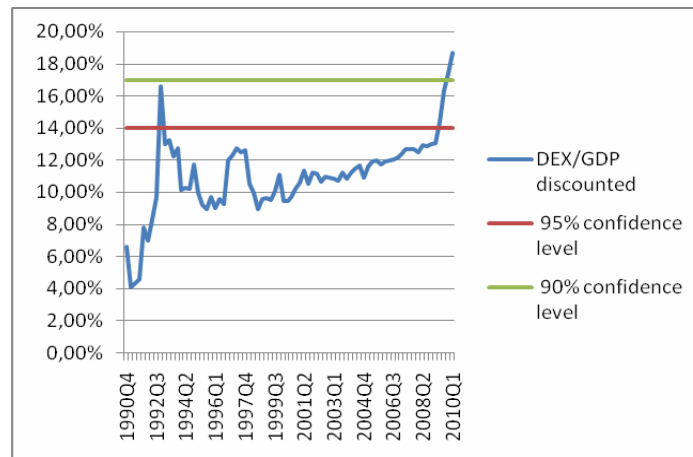
τ	$\hat{\alpha}_1(\tau)$	$t_n(\tau)$	critical value 5%	H_0	critical value 10%	H_0
10	0.9279	-3.4162	-2.6576	reject	-2.33988	Reject
11	0.9298	-3.3176	-2.3226	reject	-1.96977	Reject
12	0.9351	-3.1397	-2.4196	reject	-2.07967	Reject
13	0.9354	-3.1932	-2.6613	reject	-2.34407	Reject
14	0.9373	-3.0801	-2.5312	reject	-2.1943	Reject
15	0.9512	-2.2712	-2.4942	do not reject	-2.15427	Reject
16	0.9512	-2.5867	-2.4724	reject	-2.13249	Reject
17	0.9572	-2.6627	-2.5284	reject	-2.19114	Reject
18	0.9617	-2.1478	-2.5601	do not reject	-2.22728	do not reject

In order to identify the critical conditional quantile of the Romanian external debt, we need to test the null hypothesis $H_0: \alpha_1(\tau) = 1$ at various quantiles by using the t -ratio test $t_n(\tau)$, with the zero-mean restriction at 10% significance imposed in the ADF representation. The results in Table 4 shows that the critical quantile found using Romanian external debt data is equal to 0.17, which is the debt ceiling at a confidence level of 95%.

Also, we identified the critical conditional quantile of the Romanian external debt with significance level of 5%. The results reported in Table 4 strengthen macroeconomic conclusion, that at a confidence level of 95% the constraints are stronger than at a confidence level of 90%.

Figure 2

**Romanian Discounted External Debt/GDP Ratio
and Fixed Quantiles**



The graphic representation of the two critical conditional quantiles highlights that y_t exceeds once the 17th conditional quantile and twice the 14th conditional quantile (Figure 2).

Debt Ceiling

If y_t , the discounted external debt-GDP ratio, is a stationary zero-mean process with significance level of 10%, which is a necessary and sufficient condition for global sustainability, we use the critical conditional quantile as a debt ceiling (\bar{D}_t), following Lima L. *et al.* (2006) methodology.

The debt ceiling of the Romanian external debt-GDP ratio is constructed through in-sample forecast of the 17th and the 14th critical conditional quantiles, given by the ADF formulation:

$$\bar{D}_t = \hat{\alpha}_1(\tau_{crit})y_{t-1} + \hat{\alpha}_2(\tau_{crit})\Delta y_{t-1} + \dots + \hat{\alpha}_6(\tau_{crit})\Delta y_{t-5}$$

Figure 3 displays the in-sample path of the debt-ceiling which is nothing else but the in-sample forecast of the 17th critical conditional quantile and of the 14th critical conditional quantile.

Figure 3

**Discounted DEX/GDP Ratio and the Debt Ceiling
(Processed in R)**

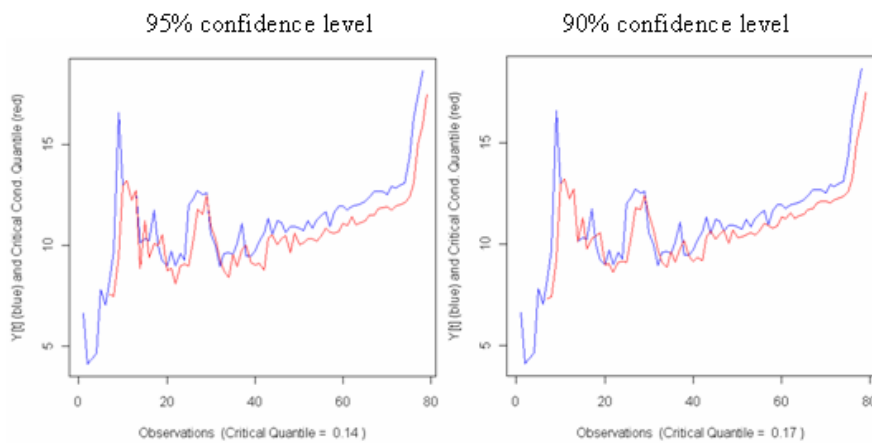


Figure 4
Undiscounted DEX/GDP Ratio, Fixed Quantiles and Debt Ceilings Turned into Undiscounted Equivalents

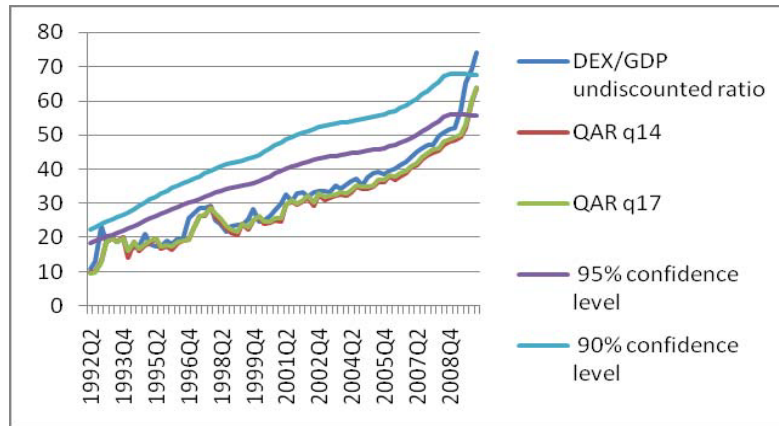
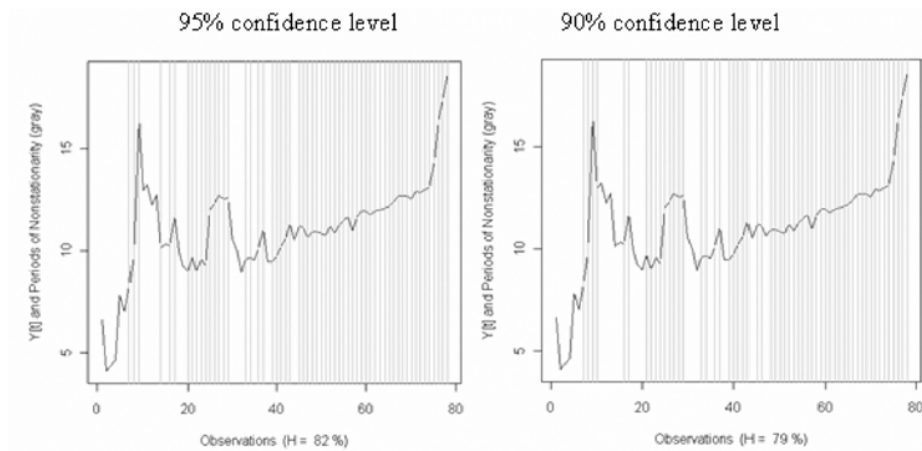


Figure 5
Episodes of Sustainable and Unsustainable Behavior (Processed in R)



In fact, the debt ceiling is nothing else than the critical conditional quantile of the discounted external debt-GDP ratio, $\bar{D}_t = Q_{y_t}(\tau_{crit}|F_{t-1})$. The proposed debt ceiling is a simple way to separate paths of external debt that are not sustainable from the ones that satisfy the long-run transversality condition (Figure 4).

Comparing both time series y_t and $Q_{y_t}(\tau_{crit}|F_{t-1})$ one can compute the statistical H, which represents the percentage of periods in which y_t exhibits a (local) nonstationary

behavior. H is the relative frequency of nonstationary periods and represents the percentage of violations of the transversality condition still compatible with long-run external debt sustainability for the in-sample forecast.

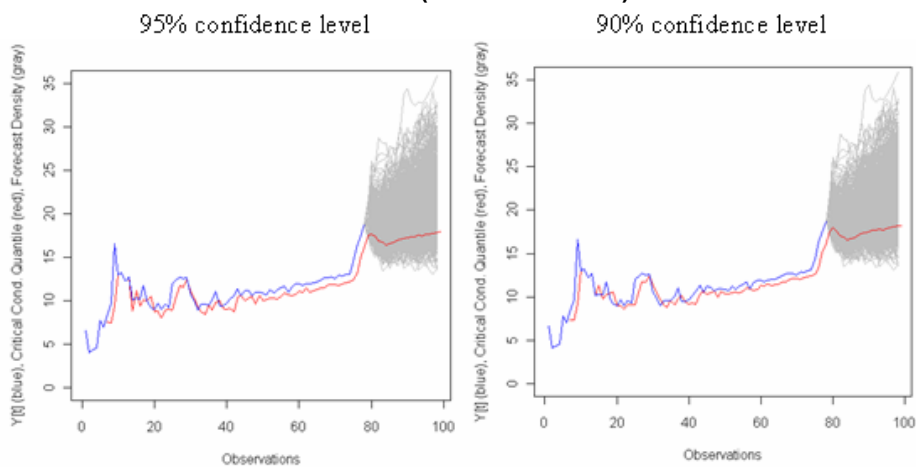
The bar in Figure 5 indicates episodes in which the external debt presented an unsustainable behavior. Despite the fact that the discounted external debt of Romania is globally sustainable with a significance level of 10%, it is obvious that in the last 5 years there was no episode of debt adjustment policies.

Next, we present the out-of-sample forecasts of the Romanian external debt, based on the methodology of the recursive generation of conditional densities of y_t , introduced by Koenker and Xiao (2004). The out-of-sample forecasts were constructed with a maximum forecast horizon $s_{max} = 20$ periods, with 1,000 trajectories for the y_t process.

The line represents the forecast debt ceiling, which is the upper trajectory that satisfies the transversality condition of the non-Ponzi scheme. As the line is not decreasing and it does not converge to zero in the long run, a decision regarding the indebtedness policy must be taken. The forecast debt ceiling will guide the decision maker to decide or not to take some action.

Figure 6

In-sample and Out-of-sample Forecast of the Critical Conditional Quantile (Processed in R)



These graphs (Figure 6) confirm the results obtained with significance level of 5%, but it contradicts the ones obtained with significance level of 10%. Even if the Romanian external debt is globally sustainable with significance level of 10%, the lower line does not show a downward slope. This happens because since March 2009 we have been the witnesses of the longest episode of local unsustainability of the Romanian external debt in the last two decade.

Putting all together, the discounted Romanian external debt is globally sustainable with a significance level of 10% despite the fact that local unsustainability can be found at some fixed quantiles. But the length of the last episode of local unsustainability through which Romania is passing may determine that the Romanian external debt will no longer be sustainable even with a significance level of 10%.

V. Conclusion

In this paper, we have empirically explored the question of whether the indebtedness policy in Romania is sustainable in the long-run using data on discounted external debt for the period from 1990Q4 to 2010Q4. Following recent econometric studies that suggest the existence of regime shifts of external debt (Yilanci, Özcan; 2008); we use a quantile autoregression model proposed by Koenker and Xiao (2004) to test if the data provides evidence of unsustainability in the Romanian external debt.

The indebtedness policies of Romania are unsustainable when debt ceiling is exceeded. In the last two decade we have witnessed two major local unsustainability episodes, identified by our model:

- in the 1992Q4 when the government nationalized the foreign currency of legal persons after a very high increase of the external debt ratio over a quarter;
- since March 2009 when in the context of the GDP decreasing the external debt increased in absolute terms by almost the same as in 2007 and 2008, which was a period marked by a GDP growth of at least 6% per year.

The Romanian external debt is not globally sustainable with significance level of 5% and is globally sustainable with significance level of 10%, even if the debt ceiling does not show a downward slope. If in the next period will preserve the macroeconomic context of the last quarters, it is possible that external debt will no more be sustainable at a level of significance of 10%.

The measure of debt ceiling presented in this paper aims to contribute to monitoring the indebtedness level, developing a "debt-warning system" that helps the macroeconomist to identify "dangerous" debt paths, deemed to be unsustainable.

The accelerated deterioration of current account imbalances, making the increased borrowing costs more expensive to cover the deficits and could threaten Romania's external debt sustainability.

Further theoretical developments of economic models that could be used to explain the existence of a band of sustainability, in addition to further empirical applications, would be very fruitful.

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