



# RATIONAL BUBBLES EXIST IN THE G-7 STOCK MARKETS? THRESHOLD COINTEGRATION APPROACH<sup>1</sup>

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## Abstract

*The purpose of this paper is to investigate whether rational bubbles exist in the G-7 stock markets during the period from January 1980 to July 2008 using the threshold cointegration approach with asymmetric adjustments advanced by Enders and Siklos (2001). The results of threshold cointegration technique reveal that rational bubbles are nonexistent in both Canadian and Japanese stock markets during the period from January 1980 to July 2008. Further, the positive deviations from values are eliminated quicker than negative deviations and the price (not the dividend) is responsible for most of the adjustments.*

**Keywords:** rational bubbles, Threshold Cointegration Test, asymmetric adjustment

**JEL Classification:** C14, C22, G12

## 1. Introduction

During the past two decades, a vast amount of research has been devoted to investigating the presence of rational bubbles in stock markets (e.g., Campbell and Shiller, 1987; Diba and Grossman, 1988a; Froot and Obstfeld, 1991; Timmermann, 1995; Crowder and Wohar, 1998; Bohl, 2003; Nasseh and Strauss, 2004; Cuñado *et al.*, 2005; Mokhtar *et al.*, 2006; Chang *et al.*, 2007, among others). The occurrence of rational bubbles signifies that no long-run relationships exist between stock price and dividend.

In pursuit of determining whether or not stock price and dividend are cointegrated, empirical studies have, for the most part, employed cointegration techniques. Among the most notable of these is the widely employed Johansen cointegration test (Johansen, 1988; Johansen and Juselius, 1990), which belongs to the linear model

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<sup>1</sup> This study was presented at 2013 Global Business, Economics, and Finance Conference, Wuhan University, China, 9-11 May 2013.

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with symmetric adjustment. However, the absence of a long-run relationship between stock price and dividend in these initial tests might be attributed to the employment of linear tests for mean reversion. There are in fact asymmetries in any adjustment toward fundamental values with respect to positive and negative shocks. Added to this, this type of test for symmetric cointegration has lower power in the case of asymmetric adjustments.

As pointed out by Balke and Fomby (1997), the power of linear cointegration tests is lower in an asymmetric adjustment process. More to the point, it is very likely that the assumption of symmetric adjustments yields poor results when it comes to equilibrium relationships because conventional cointegration tests do not take asymmetric adjustments into account. Enders and Granger (1998) also show that the standard tests for unit root and cointegration all have lower power in the presence of mis-specified dynamics. This is important, since the linear relationship is inappropriate if prices are sticky in the downward, but not in the upward direction. Madsen and Yang (1998) have provided evidence that prices are sticky in the downward direction and that such stickiness means that the adjustments of prices are asymmetric.<sup>3</sup>

Motivated by the above considerations, the purpose of this study is to re-investigate whether rational bubbles were present in the G-7 stock markets during the January 1980 to July 2008 period by using a more advanced econometric method - the threshold cointegration test proposed by Enders and Siklos (2001).

The remainder of this paper is organized as follows. Section II briefly presents the theoretical model of rational bubbles. Section III describes the data used in this study. Section IV presents the methodology. Section V discusses the empirical results, and Section VI reviews the conclusions we draw.

## **2. The Theoretical Model of Rational Bubbles**

Following Campbell *et al.* (1997), Cuñado *et al.* (2005), and Koustas and Serletis (2005), our model of net simple return on a stock is defined as

$$R_{t+1} = \frac{P_{t+1} - P_t + D_{t+1}}{P_t} = \frac{P_{t+1} + D_{t+1}}{P_t} - 1, \quad (1)$$

where:  $R_{t+1}$  denotes the stock return in period  $t+1$  and  $D_{t+1}$  is the dividend in period  $t+1$ . Taking the mathematical expectation on Equation (1), based on information available at time  $t$ , and rearranging terms, we obtain

$$P_t = E_t \left[ \frac{P_{t+1} + D_{t+1}}{1 + R_{t+1}} \right] \quad (2)$$

Solving Equation (2) forward  $k$  periods, we obtain the semi-reduced form

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<sup>2</sup> Other reasons for the asymmetric adjustment are the presence of transactions costs.

$$P_t = E_t \left[ \sum_{i=1}^k \left( \frac{1}{1+R_{t+i}} \right)^i D_{t+i} \right] + E_t \left[ \left( \frac{1}{1+R_{t+k}} \right)^k P_{t+k} \right] \quad (3)$$

In order to yield a unique solution to Equation (3), we must assume that the expected discounted value of the stock in the indefinite future converges to zero

$$\lim_{k \rightarrow \infty} E_t \left[ \left( \frac{1}{1+R_{t+k}} \right)^k P_{t+k} \right] = 0 \quad (4)$$

Further, the convergence assumption let us yield the fundamental value of the stock as the expected present value of future dividends

$$F_t = E_t \left[ \sum_{i=1}^{\infty} \left( \frac{1}{1+R_{t+i}} \right)^i D_{t+i} \right] \quad (5)$$

Getting out of the convergence assumption, Equation (4) can lead to an infinite number of solutions and any one of which can be written in the form

$$P_t = F_t + B_t \quad \text{where} \quad B_t = E_t \left[ \frac{B_{t+1}}{1+R_{t+1}} \right] \quad (6)$$

The additional term  $B_t$  is called a “rational bubble”, in the sense that it is entirely consistent with rational expectations and the time path of expected returns.

Diba and Grossman (1988b) also define a rational bubble to be a self-confirming divergence of stock prices from market fundamentals in response to extraneous variables. If the nonstationarity of dividends accounts for the nonstationarity of stock prices, then stock prices and dividends are cointegrated. The null hypothesis of rational bubbles can be tested by testing for the cointegrating relationship between dividends and stock prices. A cointegrating relationship between dividends and stock prices is inconsistent with rational bubbles. Cointegration implies that two or more time series cannot drift apart indefinitely as they must satisfy a long-run equilibrium condition.

### 3. Data

The data set consists of monthly stock market indices and dividends<sup>4</sup> for the G-7

<sup>4</sup> Index market value on Datastream is the sum of share price multiplied by the number of ordinary shares in issue for each constituent index. The amount in issue is updated whenever new tranches of stock are issued or after a capital change, and the total dividend amount for each index is the dividend yield multiplied by the total market value in this study. They are

calculated as follows:  $MV_t = \sum_1^n (P_t * N_t)$ ;  $DY_t = \sum_1^n (D_t * N_t) / \sum_1^n (P_t * N_t)$ ;

$TD_t = \sum_1^n (D_t * N_t) = DY_t * MV_t$ . Where:  $N_t$  is number of shares in issue on day t;  $P_t$  is

countries from DataStream over the period January 1980 to July 2008. The stock market indices for the G-7 countries are American S&P 500 index, Canadian TSX Composite index, French CAC 40 index, German DAX 30 index, British FTSE100 index, Japanese TOPIX 1000 index, and Italian MIB 30 Index. The monthly closing prices ( $P_t$ ) and corresponding dividend yields ( $DY_t$ ) are collected and the dividends ( $D_t$ ) for the stock market indices are calculated from the corresponding dividend yields,  $D_t = P_t \times DY_t$ . This calculation is done in accordance with prior studies (see, for example, Koivu *et al.*, 2005). Nominal stock prices and dividends were expressed in real terms using the Consumer Price Index (all items). Real stock price and dividend series were expressed in natural logarithms. The reason we choose the sample period is avoiding the subprime storm, which might cause the potential presence of structural break in the sample stock indexes studied. Table 1 reports the summary statistics of the data studied. We find that dividends for the G-7 countries all exhibit significant linear and nonlinear dependencies. The Jarque-Bera statistics also indicate that all dividends are non-normal, except for France and Italy.<sup>5</sup> The measures for skewness and excess kurtosis show that the stock market return series, except for USA and Japan, are highly leptokurtic and negatively skewed with respect to the normal distribution. The Jarque-Bera statistics indicate that all stock market returns are non-normal, with the exception of those of USA, Canada, and Japan.

#### **4. Threshold Cointegration Tests Based on Enders and Siklos (2001) Approach**

In this paper, we employ the threshold cointegration technique advanced by Enders and Siklos (2001) to test the relationship between stock price and dividend for the G-7 countries. This test involves a two-stage process. In the first stage, we estimate a long-run equilibrium relationship in the form:

$$P_t = \alpha + \beta D_t + \mu_t \quad (7)$$

Here,  $P_t$  is the stock price and  $D_t$  stands for dividend.  $\mu_t$  is the normal distributed error-term with zero expected mean, constant variance and no autocorrelation. The

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unadjusted share price on day  $t$ ;  $TD_t$  is total dividend amount on day  $t$ ;  $DY_t$  is aggregate dividend yield on day  $t$ ;  $D_t$  is dividend per share on day  $t$  and  $n$  is number of constituents in index.

<sup>5</sup> According to Jawadi (2009), in practice there is no optimal dividend policy and dividend distribution regimes are significantly different from one company to another. Besides, dividend distribution rates also vary a lot from one country to another. Normally, they can be divided into two groups of countries. The first group, with a strong average distribution rate (between 50% and 60%) is made up of Canada, the UK and the US. The second group, with a weak average distribution rate ( $\cong 30\%$ ) is made up of Germany, France, Italy, and Japan. This difference is due to the structure of capital property. Indeed, managerial companies have a higher weight in North America and the UK, while in the other countries, non-managerial companies have the highest weight.

second stage pertaining to the OLS estimates of  $\rho_1$  and  $\rho_2$  is based on the following regression:

$$\Delta u_t = I_t \rho_1 u_{t-1} + (1 - I_t) \rho_2 u_{t-1} + \sum_{i=1}^l \gamma_i \Delta u_{t-i} + \varepsilon_t \quad (8)$$

where:  $\varepsilon_t$  is a white-noise disturbance and the residuals,  $\mu_t$ , in Equation (7) are substituted into Equation (8).  $I_t$  is the Heaviside indicator function, so that  $I_t = 1$  if  $u_{t-1} \geq \tau$ , and  $I_t = 0$  if  $u_{t-1} < \tau$ , where  $\tau$  is the threshold value. The necessary condition for  $\{\mu_t\}$  to be stationary is:  $-2 < (\rho_1, \rho_2) < 0$ . If the variance of  $\varepsilon_t$  is sufficiently large, it is also possible for one value of  $\rho_j$  to range between  $-2$  and  $0$  and for the other value to be equal to zero. Although there is no convergence in the regime with the unit-root (i.e., the regime in which  $\rho_j = 0$ ), a large realization of  $\varepsilon_t$  will switch the system to the convergent regime. Enders and Granger (1998) and Enders and Siklos (2001) share the view that in either case, under the null hypothesis of no convergence, the F-statistic for the null hypothesis of  $\rho_1 = \rho_2 = 0$  has a nonstandard distribution, since the critical values for this non-standard F-statistic depend on the number of variables used in the cointegrating vector. In this study, we follow Enders and Siklos (2001) to calculate the critical values for the three-variable case. These critical values are not reported here, but are available upon request. Enders and Granger (1998) also show that if the sequence is stationary, the least square estimates of  $\rho_1$  and  $\rho_2$  have an asymptotic multivariate normal distribution.

The model using Equation (8) is referred to as the Threshold Autoregression Model (TAR), while the test for the threshold behavior of the equilibrium error is termed the threshold cointegration test. If we assume the system is convergent,  $\mu_t = 0$  can be considered the long-run equilibrium value of the sequence. If  $\mu_t$  is higher than the long-run equilibrium, the adjustment is  $\rho_1 \mu_{t-1}$ , but if  $\mu_t$  is lower than the long-run equilibrium, the adjustment is  $\rho_2 \mu_{t-1}$ . The equilibrium error, therefore, behaves like a threshold autoregressive process. The null hypothesis of  $\rho_1 = \rho_2 = 0$  tests for the cointegration relationship and if this null is rejected, then this is evidence of cointegration among the variables. When the null hypothesis of  $\rho_1 = \rho_2 = 0$  is rejected, it is worth testing further for symmetric adjustments (i.e.,  $\rho_1 = \rho_2$ ) by using a standard F-test. When adjustment is symmetric, i.e.,  $\rho_1 = \rho_2$ , Equation (8) becomes the prevalent augmented Dickey-Fuller test. Rejecting both the null hypotheses of  $\rho_1 = \rho_2 = 0$  and  $\rho_1 = \rho_2$  indicates the existence of threshold cointegration with the asymmetric adjustments.

Instead of estimating Equation (8) with the Heaviside indicator, which depends on the level of  $\mu_{t-1}$ , the decay can also be allowed to depend on the change in  $\mu_{t-1}$  in the

previous period. The Heaviside indicator can then be specified as  $I_t = 1$  if  $\Delta u_{t-1} \geq \tau$  and as  $I_t = 0$  if  $\Delta u_{t-1} < \tau$ , where  $\tau$  is the threshold value. According to Enders and Granger (1998), this model is especially valuable when adjustment is asymmetric as the series exhibits more 'momentum' in one direction than in the other. This model is called the Momentum-Threshold Autoregression (M-TAR) Model. The TAR model can capture a 'deep' cycle process if, for example, the positive deviations are more prolonged than the negative ones. The M-TAR model, on the other hand, allows the autoregressive decay to depend on  $\Delta u_{t-1}$ . Thus, the M-TAR representation is able to capture 'sharp' movements in a sequence.

In general, the value of  $\tau$  is unknown, and it must be estimated along with the values of  $\rho_1$  and  $\rho_2$ . A consistent estimate of the threshold  $\tau$  can be obtained by using Chan's (1993) method to search among possible threshold values to minimize the residual sum of squares from the fitted model. Enders and Siklos (2001) apply Chan's methodology to a Monte Carlo study to obtain the F-statistic for the null hypothesis of  $\rho_1 = \rho_2 = 0$  when they estimate threshold  $\tau$  using Chan's procedure. As there is generally no prescribed rule as to whether to use the TAR or M-TAR model, the recommendation is to select the adjustment mechanism using a model selection criterion, such as the Akaike Information criteria (AIC) or Schwartz criteria (SC).

## 5. Empirical Results

As a first step, we apply the Engle-Granger procedure to test whether rational bubbles exist in the G-7 stock market. Table 2 presents the results of the application of the Engle-Granger procedure to Equation (7) for each country, with the lag length selected based on the Akaike Information Criteria (AIC). The results of Engle-Granger cointegration test indicate that the null hypotheses of no cointegration can not be rejected for all G-7 stock markets. The rational bubbles are existent in the G-7 stock markets.

However, the absence of a long-run relationship between stock price and dividend in these initial tests might be attributed to the employment of linear tests for mean reversion. There are in fact asymmetries in any adjustment toward fundamental values with respect to positive and negative shocks. Added to this, this type of test for symmetric cointegration has lower power in the case of asymmetric adjustments. For these two reasons, we choose to use the threshold cointegration test. The results of this test with a threshold value of zero are reported in Table 3. Further, when we use the AIC model selection criterion, the M-TAR model is favored in all cases. Under these conditions, we cannot reject the null hypothesis of  $\rho_1 = \rho_2 = 0$  for all G-7 countries, which signifies that the rational bubbles were existent in the G-7 stock markets. Notably from the 2000s, the world economy was softening and a recession loomed. Moreover, after the effect of 9/11 conspiracy in 2001 that pushed the economy into recession worldwide, the government policy was to maintain low and stable inflation, and interest rates to stimulate economy. The effect was declining in the equity risk premium and increasing in productivity growth, which supports both the

higher level of prices relative to dividends. Sharpe (2001) has argued that lower inflation would lead to lower required rates of return and higher expected earnings growth, thus supporting higher stock prices. Therefore, the lower dividend yield remains at a historically low level observed in the sample period, implying overvaluations in stock prices. We conclude that long-run relationship between stock price and dividend generally fails when we assume linear adjustment or allow for asymmetric adjustments.

However, given the presence of measurement errors and/or adjustment costs, there is no reason to presume that the threshold value is equal to zero. Further, when we also use the AIC model selection criterion, the M-TAR model is also favored in all cases. As shown in Table 4, it is clearly apparent that no rational bubbles (i.e., cointegration) are existent in the Canadian and Japanese stock markets when we use Chan's method to obtain a consistent estimate of the threshold value<sup>6</sup>. Moreover, the hypothesis of a symmetric adjustment ( $\rho_1 = \rho_2$ ) is also rejected for these two countries. A major difference between these results and those in Tables 2 and 3 is that the evidence for cointegration (i.e., no rational bubbles) is substantially strengthened when we allow for asymmetries and do not presume that the threshold value is equal to zero.

Having found evidence supporting asymmetric adjustment for Canadian and Japanese stock markets, an asymmetric error-correction model can be used to investigate the movement of variables to the long-run equilibrium relationship. We estimate the following system of asymmetric error-correction model for both two countries:

$$\Delta P_t = \alpha_{10} + I_t \rho_{1P} \mu_{t-1} + (1 - I_t) \rho_{2P} \mu_{t-1} + \sum_{i=1}^k \alpha_{1i} \Delta P_{t-i} + \sum_{i=1}^k \beta_{1i} \Delta D_{t-i} + \varepsilon_{1t} \quad (9)$$

$$\Delta D_t = \alpha_{20} + I_t \rho_{1D} \mu_{t-1} + (1 - I_t) \rho_{2D} \mu_{t-1} + \sum_{i=1}^k \alpha_{2i} \Delta D_{t-i} + \sum_{i=1}^k \beta_{2i} \Delta P_{t-i} + \varepsilon_{2t} \quad (10)$$

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<sup>6</sup> One justification of our findings is that the Canadian equity market is less liquid than the U.S. market where the average firm size is much greater. Larger companies have more resources to distribute to their shareholders. In fact, Fama and French (2001) find that the probability of paying dividends increases with firm size. Market liquidity may also influence a firm's dividend payout decision. Lower liquidity leads to information asymmetry. To mitigate the adverse effects of information asymmetry, management might choose to pay higher dividends. Another reason is that monetary policy in Canada maintaining low and stable consumer price inflation is the contribution that can make to promoting economic and financial stability with the bubbles. Finally, Canadian index was characterized that adjustment is active when prices deviations from fundamental. On the other hand, Japanese firms, particularly keiretsu-member firms, face less information asymmetry and fewer agency conflicts than U.S. firms and other countries. Lower levels of information asymmetry and agency conflict in Japanese firms, suggest that dividends do not act as a signal of information or as a disciplinary mechanism, and that Japanese managers need not fear adjusting dividends in response to earnings changes. Lintner (1956) indicates that Japanese keiretsu-member firms adjust dividends more quickly than both U.S. firms and cut dividends in response to poor performance more quickly than U.S. firms. Japanese dividend policy, especially that of keiretsu-member firms, contains less information and is more responsive to performance than U.S. and other countries' dividend policy.

where:  $\mu_{t-1}$  is the residual of Equation (7),  $I_t=1$  if  $\Delta u_{t-1} \geq \tau$  and  $I_t=0$  if  $\Delta u_{t-1} < \tau$ , where  $\tau$  is the threshold value. The choice of the appropriate lag length is based on the multivariate AIC. The choice of non-zero threshold follows the same procedure outlined earlier. The estimated asymmetric error-correction models with consistent estimate of thresholds are shown in Table 5. The estimated coefficients of  $\rho_1$  and  $\rho_2$  determine the speed of adjustment for positive and negative deviations from fundamental values, respectively. If  $|\rho_1|$  is higher than  $|\rho_2|$ , it implies that the speed of adjustment for positive deviations is faster. Based on Table 5, we find that positive deviations from fundamental values are eliminated quicker than negative deviations and the price (not the dividend) is responsible for most of the adjustments. The results highlight more generally the roles played by price adjustments.

## 6. Conclusions

In this study, we revisit the issue as to the presence of rational bubbles in the G-7 stock markets during the period from January 1980 to July 2008 by using the cointegrating technique. Without taking asymmetric adjustments into account, the results of Engle-Granger cointegration test indicate that the rational bubbles are existent in the G-7 stock markets. However, when we change to the threshold cointegration approach with asymmetric adjustments advanced by Enders and Siklos (2001), we find more convincing evidence of the time series properties of the dividend and price, because it is flexible enough to capture non-linear adjustment patterns, and the results reveal that stock price adheres to dividend and rational bubbles were nonexistent in the Canadian and Japanese stock markets during the period from January 1980 to July 2008. Up to now, few studies employ the threshold cointegration approach with asymmetric adjustments to analyze whether the rational bubbles exist in the G-7 stock markets. Thus, this study might be able to provide high practical and academic contributions in this line of research.

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Table 1

## Summary Statistics of the Data

Panel A. $\ln D$							
Statistic	SP500	TSX	CAC40	DAX30	FTSE100	TOPIX1000	MIB30
Mean	13.086	9.636	10.211	9.521	10.502	15.215	9.289
Std. Dev.	0.316	0.784	0.259	0.232	0.161	0.342	0.752
Max.	14.444	11.989	10.729	10.570	10.712	15.715	10.063
Min.	9.738	9.382	9.718	9.087	10.226	14.634	8.297
Skewness	0.237	0.607	0.523	0.127	0.439	0.818	-0.148
Kurtosis	1.419	2.967	2.789	2.539	1.763	1.929	2.355
Jarque-Bera	11.015**	14.414***	5.523	9.975**	8.233**	12.615***	1.387
Ljung-Box Q(5)	245.47***	416.72***	302.73***	345.71***	432.05***	407.12***	372.44***
Ljung-Box Q(10)	345.66***	703.58***	454.22***	525.93***	721.16***	706.27***	633.49***
Ljung-Box Q2(5)	525.27***	416.20***	351.11***	393.93***	433.46***	406.39***	381.26***
Ljung-Box Q2(10)	644.89***	654.11***	426.49***	461.02***	736.63***	712.23***	612.65***
Panel B. $\Delta \ln P$							
Statistic	SP500	TSX	CAC40	DAX30	FTSE100	TOPIX1000	MIB30
Mean	0.002	0.068	0.003	-0.001	0.006	0.002	0.006
Std. Dev.	0.038	0.042	0.054	0.065	0.039	0.047	0.050
Max.	0.102	0.108	0.138	0.189	0.084	0.115	0.122
Min.	-0.092	-0.123	-0.177	-0.246	-0.174	-0.229	-0.172
Skewness	-0.333	-0.418	-0.451	-0.644	-0.914	-0.139	-1.186
Kurtosis	3.212	3.146	3.833	5.158	4.409	2.884	6.738
Jarque-Bera	1.921	3.017	9.762***	29.143***	23.022***	0.338	50.103***
Ljung-Box Q(5)	1.813	4.448	2.171	4.055	3.177	6.431	5.352
Ljung-Box Q(10)	5.325	5.435	6.039	9.122	6.115	7.274	7.482
Ljung-Box Q2(5)	33.225***	3.915	11.162*	22.974***	7.842	5.354	3.122
Ljung-Box Q2(10)	50.284***	11.967	28.085***	36.282***	19.867*	8.994	9.017
Notes: 1. *, ** and *** indicate significance at the 10%, 5% and 1% levels, respectively.							
2. $\Delta \ln P = \ln P_t - \ln P_{t-1}$ , $\ln D = \ln D_t$ .							

<b>Results of Engle-Granger Cointegration Test</b>				
<i>Country</i>	<i>Stock Market Index</i>	$\rho$	<i>AIC</i>	<i>Lags</i>
USA	SP500	-0.065 (-1.873)	-3.084	0
Canada	TSX	-0.043 (-1.614)	-3.439	0
France	CAC40	-0.057 (-1.651)	-2.782	0
Germany	DAX30	-0.093 (-2.529)	-2.424	0
UK	FTSE100	-0.055 (-1.673)	-3.621	0
Japan	TOPIX1000	-0.084 (-2.396)	-3.291	1
Italy	MIB30	-0.054 (-1.223)	-3.848	0

Note: For the third column, the critical values of the t-statistics for the null hypothesis  $\rho = 0$  with the two variables in the cointegrating relationship are -3.50, -2.89 and 2.58 at the 1%, 5% and 10% significance levels, respectively.

<b>Estimated Asymmetric Adjustment Equations Using the Threshold Cointegration Test with <math>\tau = 0</math></b>							
<i>Index</i>	$\rho_1$	$\rho_2$	$\Phi_\mu$	$\rho_1 = \rho_2$	<i>AIC</i>	<i>Flag</i>	<i>Lags</i>
SP500	-0.083	-0.063	1.719	0.056	-170.754	M-TAR	4
TSX	-0.112*	-0.066	2.723	0.322	-162.213	M-TAR	4
CAC40	-0.048	-0.046	1.161	0.001	-99.324	M-TAR	4
DAX30	-0.110	-0.104	2.914	0.005	-64.411	M-TAR	4
FTSE100	-0.052	-0.015	1.415	0.438	-175.125	M-TAR	4
TOPIX1000	-0.126**	-0.033	4.373	2.354	-143.151	M-TAR	4
MIB30	0.003	-0.084**	2.262	1.993	-129.122	M-TAR	4

Notes:

- For the fourth column, this F statistic for the null hypothesis of  $\rho_1 = \rho_2 = 0$  follows a non-standard distribution; the critical values tabulated at Table 1 of Enders and Siklos (2001) are 5.2, 6.2, and 8.46 at the 1%, 5% and 10% significance level respectively.
- For the fifth column, the critical values of the standard F-statistic for the null hypothesis  $\rho_1 = \rho_2$  with the two variables in symmetric adjustment are 2.765, 3.953, and 6.943 the 1%, 5% and 10% significance levels respectively.
- \*, \*\* and \*\*\* indicate significance at the 10%, 5% and 1% levels respectively.

**Table 4**  
**Estimated Adjustment Equations Using the Threshold Cointegration Test**  
**with a Consistent Estimate of the Threshold Value of  $\tau$**

Index	$\rho_1$	$\rho_2$	$\Phi_{\mu}^*$	$\rho_1 = \rho_2$	AIC	Flag	$\tau$	Lags
SP500	-0.0532	-0.8574**	3.4342	5.4323**	-134.3486	M-TAR	-0.09736	4
TSX	-0.2793***	0.0117	7.7847**	8.5738***	-178.2181	M-TAR	0.13523	4
CAC40	-0.0336	-0.4164***	3.7218	9.4223***	-114.1393	M-TAR	-0.09443	4
DAX30	-0.0828**	-0.4201**	3.1321	3.3381*	-65.9836	M-TAR	-0.17449	4
FTSE100	-0.0573	-0.3348*	2.7895	2.1291	-199.8326	M-TAR	-0.09437	4
TOPIX1000	-0.1394***	0.2663**	9.1219**	9.7558***	-154.2205	M-TAR	-0.08672	4
MIB30	-0.0357	-0.2643**	1.9757	4.8244*	-112.1223	M-TAR	-0.15436	4

Notes: For the fourth column, this F statistics for the null hypothesis of  $\rho_1 = \rho_2 = 0$  follows a non-standard distribution; the critical values tabulated at Table 5 of Enders and Siklos (2001) are 5.52, 6.56, and 8.91 at the 1%, 5% and 10% significance level respectively. For the fifth column, the critical values of the standard F-statistic for the null hypothesis  $\rho_1 = \rho_2$  with the two variables in symmetric adjustment are 2.765, 3.953, and 6.943 at the 1%, 5% and 10% significance levels, respectively.  
 3. \*, \*\* and \*\*\* indicate significance at the 10%, 5% and 1% levels, respectively.

Table 5									
The Estimated Asymmetric Error-correction Models									
Index	Dependent Variable	$\sum_{i=1}^k \alpha_i$	$\sum_{i=1}^k \beta_i$	Lags	$\rho_1$	$\rho_2$	$\tau$	Ljung-Box Q-Stat.	
								Q(2)	Q <sup>2</sup> (2)
TSX	$\Delta p_t$	0.187	-0.012	3	-0.137*** (-3.105)	0.034 (0.998)	0.015	0.939	0.624
	$\Delta d_t$	0.234	0.203	4	-0.055* (-1.822)	0.051 (1.176)	-0.068	0.107	0.141
TOPIX 1000	$\Delta p_t$	0.321	0.222	3	-0.222*** (-3.924)	0.211 (1.504)	-0.075	0.766	1.663
	$\Delta d_t$	0.158	-0.123	2	0.082** (2.733)	-0.005 (-0.218)	0.094	0.125	1.172

Notes:

- The t-statistics are in parentheses.
- \*, \*\* and \*\*\* indicate significance at the 10%, 5% and 1% levels, respectively.
- Asymmetric Error-Correction Models:

$$\Delta P_t = \alpha_{10} + I_t \rho_{1P} \mu_{t-1} + (1 - I_t) \rho_{2P} \mu_{t-1} + \sum_{i=1}^k \alpha_{1i} \Delta P_{t-i} + \sum_{i=1}^k \beta_{1i} \Delta D_{t-i} + \varepsilon_{1t}$$

$$\Delta D_t = \alpha_{20} + I_t \rho_{1D} \mu_{t-1} + (1 - I_t) \rho_{2D} \mu_{t-1} + \sum_{i=1}^k \alpha_{2i} \Delta D_{t-i} + \sum_{i=1}^k \beta_{2i} \Delta P_{t-i} + \varepsilon_{2t}$$

where:  $\mu_{t-1}$  is the residual from Eq. (7),  $I_t = 1$  if  $\Delta u_{t-1} \geq \tau$  and  $I_t = 0$  if  $\Delta u_{t-1} < \tau$ , where  $\tau$  is the threshold value. The choice of the appropriate lag length is based on the multivariate AIC.