3. THE RELATIONSHIP BETWEEN ENERGY CONSUMPTION AND OUTPUT: A FREQUENCY DOMAIN APPROACH[1]

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Abstract

This paper examines the causal relationship between energy consumption and economic growth in three African countries (i.e., Algeria, Egypt, and South Africa) from 1970 to 2011 using the frequency domain-based Granger causality test proposed by Lemmens et al. (2008). Our empirical results reveal a unidirectional causality, from energy consumption to economic growth, for Algeria, bi-directional causality between energy consumption and economic growth for Egypt, and no causality in any direction between energy consumption and economic growth for South Africa. Our findings provide important implications for energy policies and strategies in the three African countries.

Keywords: energy consumption, output, Granger causality, frequency-domain, African countries

JEL Classification: C33, E23, F40

I. Introduction

The relationship between energy consumption and economic growth has been extensively studied over the past decades. These studies have tested four different hypotheses, namely growth, conservation, feedback, and neutrality hypotheses (see Payne, 2010; Ozturk, 2010). The growth hypothesis is a one-way Granger causality (GC) running from energy consumption to economic growth, whereas the conservation hypothesis is a one-way GC running from economic growth to energy consumption. The feedback hypothesis is a two-way GC between energy consumption and economic growth. As for the neutrality hypothesis, no causality exists between energy consumption and economic growth. On the one hand, this hypothesis considers energy consumption as a relatively minor component of the overall output, and thus it may have little or no impact on real GDP. On the other hand, it envisages energy

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conservation policies as incapable of adversely affecting the real GDP. Testing these four hypotheses has several important implications for energy policy and strategy. If the relationship between energy consumption and economic growth supports the growth and feedback hypotheses, excessive energy protection and a reduction in energy consumption may lead to a negative economic growth. By contrast, the conservation hypothesis asserts that energy conservation policies designed to reduce energy consumption and waste may not have an adverse impact on the real GDP. Thus, knowledge of the causal relationship and the direction between energy consumption and economic growth are of particular importance to policy makers in the process of developing appropriate energy strategies.

The vast majority of existing empirical research has focused on the time domain approach in investigating the causal relationship between energy consumption and economic growth of the developing and developed countries. In this regard, no existing study has used the frequency domain. Traditional approaches to GC have yielded many interesting insights, but they tacitly ignore the possibility that the strength and/or direction of the GC relationship, if any, could vary over different frequencies (Lemmens *et al*., 2008). The idea of disentangling the GC relationship further between two time series was first suggested by Granger (1969). He believes that a spectral density approach would give a better and more complete picture than a one-shot GC measure that is supposed to apply across all periodicities (e.g., in the short run, over the business cycle frequencies, and in the long run).

This paper presents our attempts to re-investigate the causal relationship between energy consumption and economic growth in three African countries for the first time using the frequency domain-based GC test proposed by Lemmens *et al*. (2008). This current research hopes to fill the existing gap in the literature.

This paper is organized as follows. Section 2 discusses the data and outlines the GC methodology over the spectrum proposed by Lemmens *et al*. (2008). Section 3 investigates the causal relations between energy consumption and output in the frequency domain. Section 4 concludes the paper.

II. Data and Methodology

A. The data

The annual data used in this study cover the period from 1970 to 2011 for three African countries (i.e., Algeria, Egypt, and South Africa).[3] The variables in this study include primary energy consumption (EC) and per capita real GDP (PRGDP). The EC variables are expressed in terms of million tons and obtained from the BP Statistical Review of World Energy (2012). The PRGDP, measured in constant 2005 U.S. dollars, is obtained from the World Development Indicators (WDI, 2012). Tables 1 and 2, which show the summary statistics of the PRGDP and EC, respectively, reveal that South Africa and Egypt have the highest and lowest mean PRGDP, at US$ 4,800.97 and US$ 974.52, respectively. Moreover, South Africa and Algeria have the highest

[3] *Only three African countries could be employed in this study because of the limitation of the database and the lack of data source.*

and lowest of EC, at 82.81 and 23.24 millions of tons, respectively. Most of the data series are approximately normal, with the exception of the PRGDP for South Africa.

Table 1

Summary Statistics of GDP

Country	Mean	Max.	Min.	Std. Dev.	Skew.	Kurt.	J.-B.
Algeria	2726.25	3400.00	1796.31	345.49	-0.17	3.13	0.22
Egypt	974.52	1630.93	484.35	334.17	0.24	2.14	1.72
South Africa	4800.97	6084.00	4093.76	485.94	1.00	3.48	7.42**

*Note: The sample period is from 1970 to 2011. ** indicate significance at the 5% level, respectively. The Jarque–Bera (J.-B) test is a normal test that follows a chi-squared distribution.*

Table 2

Summary Statistics of Energy Consumption (Unit: million tons)

Country	Mean	Max.	Min.	Std. Dev.	Skew.	Kurt.	J.-B.
Algeria	23.24	40.90	3.16	10.77	-0.59	2.33	3.16
Egypt	36.76	82.60	7.65	21.86	0.49	2.28	2.63
South Africa	82.81	126.30	35.93	26.88	-0.23	1.85	2.69

Note: The sample period is from 1970 to 2011. The Jarque–Bera (J.-B) test is a normal test that follows a chi-squared distribution.

Table 3 reports the results of three conventional unit root tests, such as the augmented Dickey–Fuller test, the Phillips–Peron test, and the Kwiatkowski–Phillips–Schmidt–Shin tests. The results in Table 3 show that all the variables are non-stationary in levels but are stationary in first difference.

Table 3

Univariate Unit Root Tests

Country	Level			1st difference		
	ADF	PP	KPSS	ADF	PP	KPSS
Algeria – PRGDP	-3.235(1)**	-1.324(2)	0.457[5]*	-9.208(0)***	-8.276(4)***	0.160[2]
Egypt- PRGDP	-0.991(0)	-0.902(2)	0.798[5]***	-3.822(0)***	-3.349(5)**	0.117[1]
S. Africa – PRGDP	-0.037(1)	0.305(3)	0.436[3]*	-3.686(0)**	-3.674(2)**	0.239[5]
Algeria – EC	-1.831(1)	-2.596(3)	0.651[5]***	-3.141(1)***	-3.693(4)***	0.249[5]
Egypt – EC	-2.651(1)*	-2.247(2)	0.785[5]***	-4.635(0)***	-4.731(2)***	0.334[3]
S. Africa - EC	-2.630(0)*	-2.476(2)	0.774[5]***	-4.907(0)***	-4.907(0)***	0.343[2]

B. The GC Test based on Frequency Domain[4]

Analyzing time series in the frequency domain, that is, spectral analysis, could be helpful in supplementing the information obtained by the time domain analysis (Granger, 1969; Priestley, 1981). Spectral analysis highlights the cyclical properties of data. In our study, we follow the bivariate GC test over the spectrum proposed by Lemmens *et al*. (2008), who reconsidered the original framework proposed by Pierce (1979) and proposed a testing procedure for Pierce's spectral GC measure. This GC test in the frequency domain relies on a modified version of the coefficient of coherence, which they estimated in a nonparametric fashion, and for which they derive the distributional properties.

[4] *This section of methodology follows Lemmens* et al*. (2008).*

Let X_t and Y_t be two stationary time series of length T. The goal is to test whether X_t Granger causes Y_t at a given frequency λ. Pierce's measure for GC (Pierce, 1979) in the frequency domain is performed on the univariate innovations series, u_t and v_t, derived from filtering the X_t and Y_t as univariate ARMA processes, that is,

$$\Theta^x(L)X_t = C^x + \Phi^x(L)u_t \quad (1)$$

$$\Theta^y(L)Y_t = C^y + \Phi^y(L)v_t \quad (2)$$

where: $\Theta^x(L)$ and $\Theta^y(L)$ are autoregressive polynomials, $\Phi^x(L)$ and $\Phi^y(L)$ are moving average polynomials, and C^x and C^y are potential deterministic components. The obtained innovation series u_t and v_t, which is white-noise processes with zero mean, possibly correlate with each other at different leads and lags. The innovation series u_t and v_t are the series of importance in the GC test proposed by Lemmens *et al*. (2008)[5].

Let $S_u(\lambda)$ and $S_v(\lambda)$ be the spectral density functions or spectra of u_t and v_t at frequency $\lambda \in]0,\pi[$, which is defined by

$$S_u(\lambda) = \frac{1}{2\pi}\sum_{k=-\infty}^{\infty}\gamma_u(k)e^{-i\lambda k} \quad (3)$$

$$S_v(\lambda) = \frac{1}{2\pi}\sum_{k=-\infty}^{\infty}\gamma_v(k)e^{-i\lambda k} \quad (4)$$

where: $\gamma_u(k)$ = Cov(u_t, u_{t-k}) and $\gamma_v(k)$ = Cov(v_t, v_{t-k}) represent the autocovariances of u_t and v_t at lag k. The idea of the spectral representation is that each time series may be decomposed into a sum of uncorrelated components, each related to a particular frequency, λ[6]. The spectrum can be interpreted as a decomposition of the series variance by frequency. The portion of the series variance occurring between any two frequencies is given by the area under the spectrum between those two frequencies. In other words, the area under $S_u(\lambda)$ and $S_v(\lambda)$ between any two frequencies λ and $\lambda + d\lambda$ gives the portion of variance of u_t and v_t, respectively, because of cyclical components in the frequency band (λ, $\lambda + d\lambda$).

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[5] *In the Granger-Sims causality test popularized by Sims (1972), the joint behavior of time series is described as follows: a variable X will Granger-cause the variable Y if the set of correlations between current innovations in Y and lagged innovations in X is significant.*

[6] *The frequencies λ_1, λ_2, ….., λ_N are specified as follows: $\lambda_1 = 2\pi/T$; $\lambda_2 = 4\pi/T$, .*

The highest frequency considered is $\lambda_N = 2N\pi/T$.

where: $N \equiv T/2$ if T is an even number and $N \equiv (T-1)/2$ if T is an odd number (see Hamilton, p. 159, 1994).

The cross spectrum represents the cross covariogram of two series in the frequency domain. It allows the relationship between two time series to be determined as a function of the frequency. Let $S_{uv}(\lambda)$ be the cross spectrum between the u_t and v_t series. The cross spectrum is a complex number defined as

$$S_{uv}(\lambda) = C_{uv}(\lambda) + iQ_{uv}(\lambda) = \frac{1}{2\pi}\sum_{k=-\infty}^{\infty}\gamma_{uv}(k)e^{-i\lambda k} \tag{5}$$

where: $C_{uv}(\lambda)$ called the cospectrum, and $Q_{uv}(\lambda)$, called the quadrature spectrum are the real and imaginary parts, respectively, of the cross spectrum, and $i = \sqrt{-1}$. Here, $\gamma_{uv}(k) = \mathrm{Cov}(u_t, v_{t-k})$ represents the cross-covariance of u_t and v_t at lag k. The cospectrum $Q_{uv}(\lambda)$ between two series u_t and v_t at frequency λ can be interpreted as the covariance between two series u_t and v_t that is attributable to cycles with frequency λ. The quadrature spectrum looks for evidence of out-of-phase cycles (Hamilton, 1994). The cross spectrum can be estimated non-parametrically by

$$\hat{S}_{uv}(\lambda) = \frac{1}{2\pi}\left\{\sum_{k=-M}^{M} w_k \hat{\gamma}_{uv}(k)e^{-i\lambda k}\right\} \tag{6}$$

with the empirical cross-covariances $\hat{\gamma}_{uv}(k) = \hat{COV}(u_t, v_{t-k})$ and with window weights w_k for $k = -M,..,M$. Equation (6) is called the *weighted covariance estimator*, and the weights w_k are selected as the Bartlett weighting scheme, that is, $1 - |k|/M$. The constant M determines the maximum lag order considered. The spectra of equations (3) and (4) are estimated in a similar manner. This cross spectrum allows us to compute the coefficient of coherence $h_{uv}(\lambda)$ defined as

$$h_{uv}(\lambda) = \frac{|S_{uv}(\lambda)|}{\sqrt{S_u(\lambda)S_v(\lambda)}} \tag{7}$$

Coherence can be interpreted as the absolute value of a frequency-specific correlation coefficient. The squared coefficient of coherence involves an interpretation similar to the R-squared in a regression context. Coherence thus takes values between 0 and 1. Lemmens *et al*. (2008) showed that under the null hypothesis, wherein $h_{uv}(\lambda) = 0$, the estimated squared coefficient of coherence at frequency λ, with $0 < \lambda < \pi$ when appropriately rescaled, converges to a chi-squared distribution with two degrees of freedom[7] denoted by χ^2_2.

[7] *For the endpoints $\lambda = 0$ and $\lambda = \pi$, only one has one degree of freedom, because the imaginary part of the spectral density estimates is cancelled out.*

$$2(n-1)\hat{h}^2_{uv}(\lambda) \xrightarrow{d} \chi^2_2 \quad (8)$$

where: $\rightarrow^d$ stands for convergence in distribution, with $n = T/\left(\sum_{k=-M}^{M} w_k^2\right)$. The null hypothesis $h_{uv}(\lambda) = 0$ versus $h_{uv}(\lambda) > 0$ is then rejected if

$$\hat{h}_{uv}(\lambda) > \sqrt{\frac{\chi^2_{2,1-\alpha}}{2(n-1)}} \quad (9)$$

with $\chi^2_{2,1-\alpha}$ being the $1-\alpha$ quantile of the chi-squared distribution with 2 degrees of freedom.

The coefficient of coherence in equation (7) gives a frequency-by-frequency measure of the strength of the linear association between two time series, but it does not provide any information on the direction of the relationship between two processes. Lemmens *et al*. (2008) have decomposed the cross spectrum (equation 5) into three parts: (i) $S_{u \Leftrightarrow v}$, the instantaneous relationship between u_t and v_t; (ii) $S_{u \Rightarrow v}$, the directional relationship between v_t and the lagged values of u_t; and (iii) $S_{v \Rightarrow u}$, the directional relationship between u_t and the lagged values of v_t, that is,

$$S_{uv}(\lambda) = [S_{u \Leftrightarrow v} + S_{u \Rightarrow v} + S_{v \Rightarrow u}] = \frac{1}{2\pi}\left[\gamma_{uv}(0) + \sum_{k=-\infty}^{-1} \gamma_{uv}(k)e^{-i\lambda k} + \sum_{k=1}^{\infty} \gamma_{uv}(k)e^{-i\lambda k}\right] \quad (10)$$

The proposed spectral measure of the GC is based on the key property that u_t does not Granger-cause v_t if and only if $\gamma_{uv}(k) = 0$ for all $k < 0$. The goal is to test the predictive content of u_t relative to v_t, which is given by the second part of equation (10), that is,

$$S_{u \Rightarrow v}(\lambda) = \frac{1}{2\pi}\left[\sum_{k=-\infty}^{-1} \gamma_{uv}(k)e^{-i\lambda k}\right] \quad (11)$$

The Granger coefficient of coherence is then given by

$$h_{u \Rightarrow v}(\lambda) = \frac{|S_{u \Rightarrow v}(\lambda)|}{\sqrt{S_u(\lambda)S_v(\lambda)}} \quad (12)$$

Therefore, in the absence of GC, $h_{u \Rightarrow v}(\lambda) = 0$ for every λ in $]0, \pi[$. The Granger coefficient of coherence takes values between 0 and 1 (Pierce, 1979) and, at frequency λ, is estimated by

$$\hat{h}_{u\Rightarrow v}(\lambda) = \frac{\left|\hat{S}_{u\Rightarrow v}(\lambda)\right|}{\sqrt{\hat{S}_u(\lambda)\hat{S}_v(\lambda)}} \tag{13}$$

with $\hat{S}_{u\Rightarrow v}(\lambda)$ as in equation (6), but with all weights w_k =0 for $k \geq 0$. The distribution of the estimator of the Granger coefficient of coherence is derived from the distribution of the coefficient of coherence in equation (8). Under the null hypothesis of $\hat{h}_{u\Rightarrow v}(\lambda) = 0$, the distribution of the squared estimated Granger coefficient of coherence at frequency λ, with $0 < \lambda < \pi$ is given by

$$2(n'-1)\hat{h}^2_{uv}(\lambda) \xrightarrow{d} \chi^2_2 \tag{14}$$

where: n is now replaced by $n' = T/\left(\sum_{k=-M}^{-1} w_k^2\right)$. As w_k's with a positive index k, are set to zero when computing $\hat{S}_{u\Rightarrow v}(\lambda)$, only the w_k with negative indices are taken into account. The null hypothesis $\hat{h}_{u\Rightarrow v}(\lambda) = 0$ versus $\hat{h}_{u\Rightarrow v}(\lambda) > 0$ is then rejected if

$$\hat{h}_{u\Rightarrow v}(\lambda) > \sqrt{\frac{\chi^2_{2,1-\alpha}}{2(n'-1)}} \tag{15}$$

The Granger coefficient of coherence is then computed using equation (13); the significance of causality is tested by equation (15).

III. Empirical Findings and Policy Implications

The six differentiated series of the PRGDP and EC were filtered using ARMA models to obtain the innovation series.[8] The GC results between energy consumption and output is presented below using the innovation series for the PRGDP and EC after ARMA. The lag length[9] $M = \sqrt{T}$ was used. The frequency (λ) on the horizontal axis can be translated into a cycle or periodicity of T years by $T = 2\pi/\lambda$, where T (equal to 42 in our case) is the period. After filtering the series using ARMA and adjusting for lags, we were left with 34 observations. Therefore, we could consider $N = 17$ cycles of different frequencies, with the shortest possible cycle of two years and the longest cycle of 34 years.

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[8] *As the time domain- and frequency domain-based GC tests require stationary time series, we first differentiate all data series before testing.*

[9] *Following Diebold (2001), we take.*

The results of the GC tests in the frequency domain are presented in Figures 1 to 6. These figures report the Granger coefficients of coherence along with their 5% critical values (horizontal line parallel to the frequency axis) for all frequencies in the interval of $(0, \pi)$. Figure 1 presents the result of the Granger coefficient of coherence for causality running from output to energy consumption for Algeria. At 5% level of significance, the output does not Granger-cause energy consumption at either lower or higher frequencies. Figure 2 presents the result of the Granger coefficient of coherence for causality running from energy consumption to output for Algeria. At 5% level of significance, energy consumption Granger-causes output at both lower and higher frequencies. The causality running from energy consumption to output is significant between the frequencies corresponding to 1.99 and 2.29-year cycle, equivalent to a short-run period, and between the frequencies corresponding to 5.99 and 12.59-year cycle, equivalent to a long-run period. These results indicate that energy consumption Granger-causes output in the short and long run for Algeria. The Granger coefficient of coherence suggests that the causality running from energy consumption to output between frequencies corresponding to 1.99 to 2.29-year cycle is relatively weak compared with frequencies corresponding to 5.99 to 12.59-year cycle. The peak of the Granger coefficient of coherence is reached at the frequency corresponding to 7.41 years. Based on the results in Figures 1 and 2, we can conclude that energy consumption provides significant predictive power for future output movements and supports the growth hypothesis. Therefore, economic growth is dependent on energy consumption, which implies that negative energy shocks and energy conservation policies may depress economic growth in Algeria.

The horizontal line represents the critical value for the null hypothesis at the 5% level of significance for Algeria.

Figure 1

Frequency Domain GC running from Output to Energy Consumption

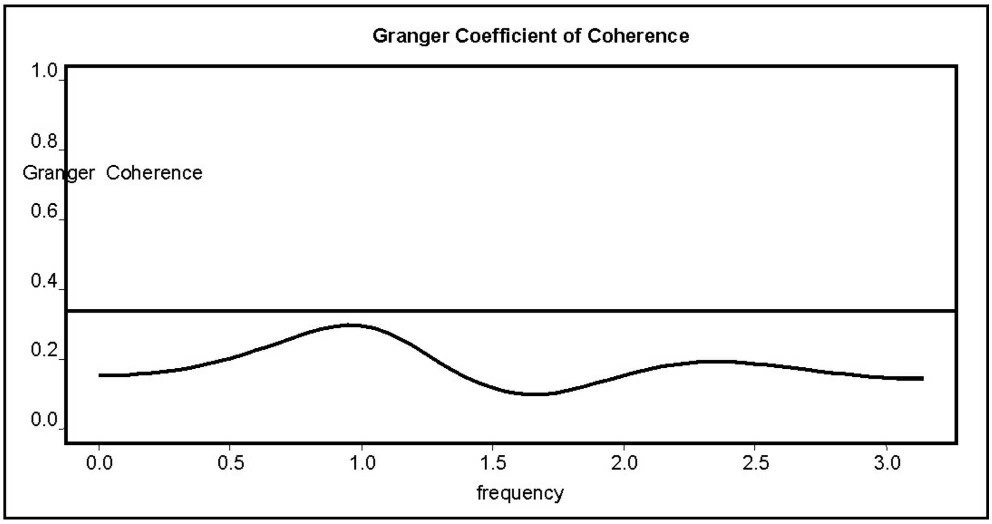


Figure 2

Frequency Domain GC running from Energy Consumption to Output

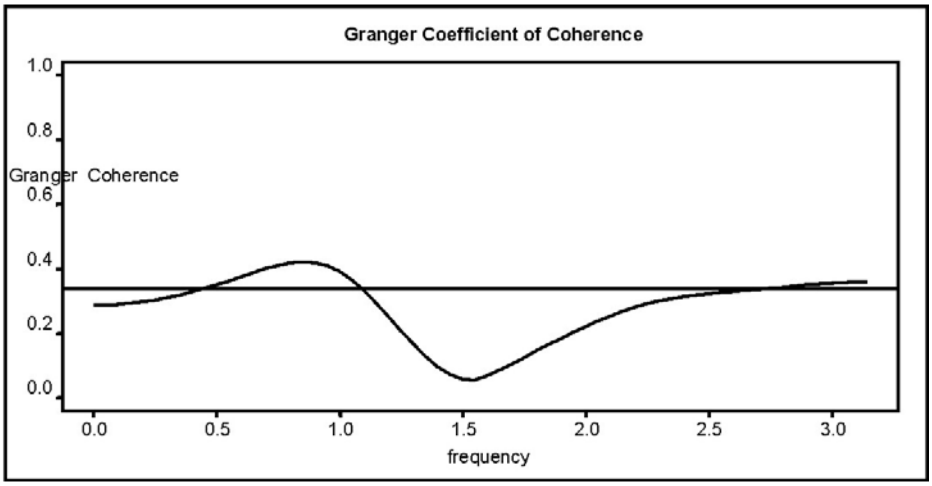


Figure 3 presents the Granger coefficient of coherence for causality running from output to energy consumption for Egypt. The figure suggests that the estimated Granger coefficient of coherence is significant between frequencies corresponding to 1.99 and 9.69-year cycle. The Granger coefficient of coherence suggests that the causality running from output to energy consumption between frequencies corresponding to 1.99 and 9.69-year cycle is relatively strong. The peak of the Granger coefficient of coherence is reached at the frequency corresponding to 3.23 years. Figure 4 presents the result of the Granger coefficient of coherence for causality running from energy consumption to output for Egypt.

The horizontal line represents the critical value for the null hypothesis at the 5% level of significance for Egypt.

Figure 3

Frequency Domain GC running from Output to Energy Consumption

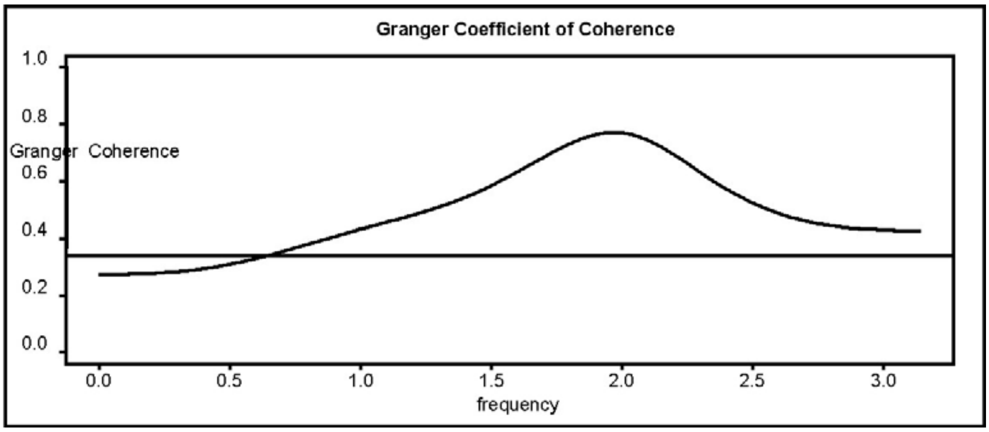
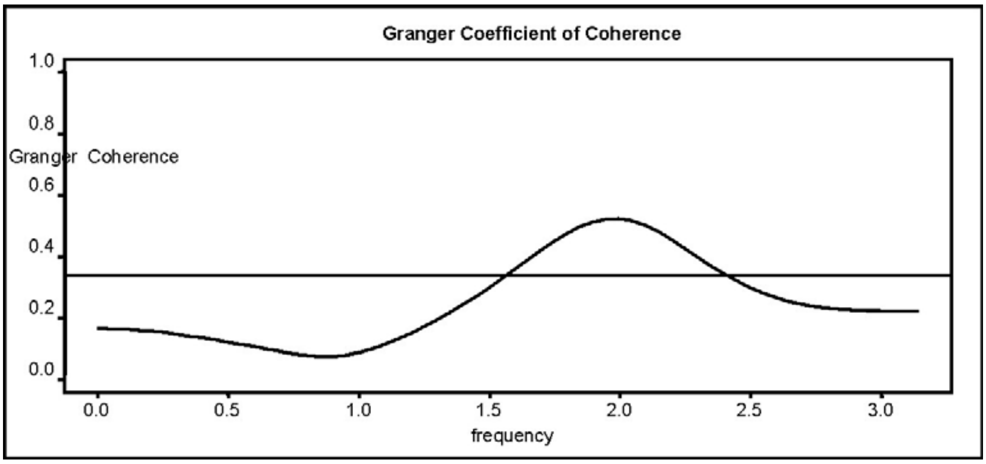


Figure 4

Frequency Domain GC running from Energy Consumption to Output.



The figure shows that the estimated Granger coefficient of coherence is significant between frequencies corresponding to 2.62 and 3.94-year cycle at 5% level of significance. The Granger coefficient of coherence suggests that the causality running from output to energy consumption between frequencies corresponding to 2.62 and 3.94 years is also relatively strong. The peak of the Granger coefficient of coherence is reached at the frequency corresponding to 3.15 years. Based on the results from both Figures 3 and 4, we can conclude that energy consumption (output) provides significant predictive power for future output (energy consumption) movements and supports the feedback hypothesis. A feedback relation between energy consumption and output indicates that excessive energy protection and reduced energy consumption may lead to pressure on economic activity in Egypt.

Figure 5 presents the result of the Granger coefficient of coherence for causality running from output to energy consumption for South Africa. The figure shows that the output does not Granger-cause energy consumption at either lower or higher frequencies at 5% level of significance. Figure 6 presents the result of the Granger coefficient of coherence for causality running from energy consumption to output for South Africa. Again, at 5% level of significance, energy consumption does not Granger-cause output at either lower or higher frequencies. Based on the results from Figures 5 and 6, we can conclude that energy consumption and output does not Granger-cause each other, supporting the neutrality hypothesis. If the neutrality between energy consumption and output holds, then this will allow policy makers to develop energy policies that are not dependent on economic activity in South Africa.

The horizontal line represents the critical value for the null hypothesis at the 5% level of significance for South Africa

Figure 5

Frequency Domain GC running from Output to Energy Consumption.

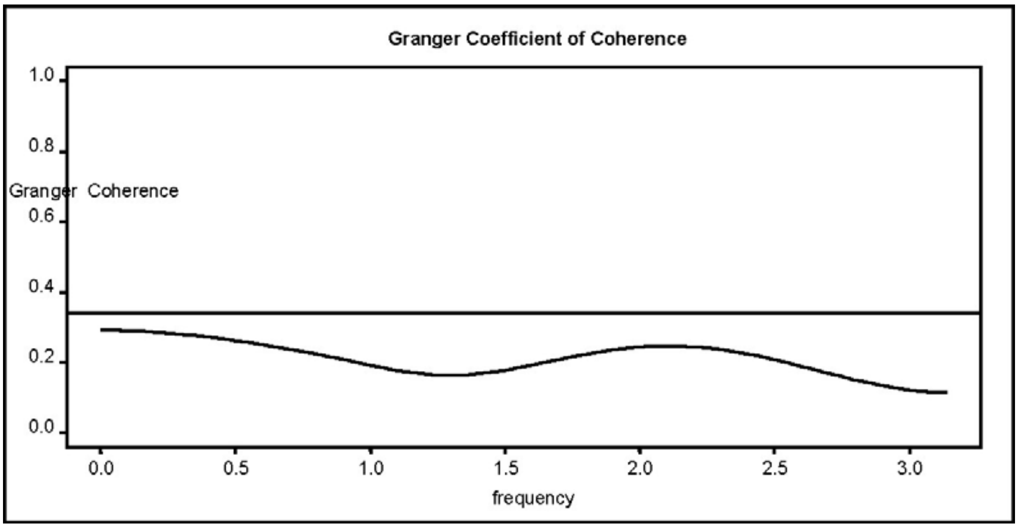
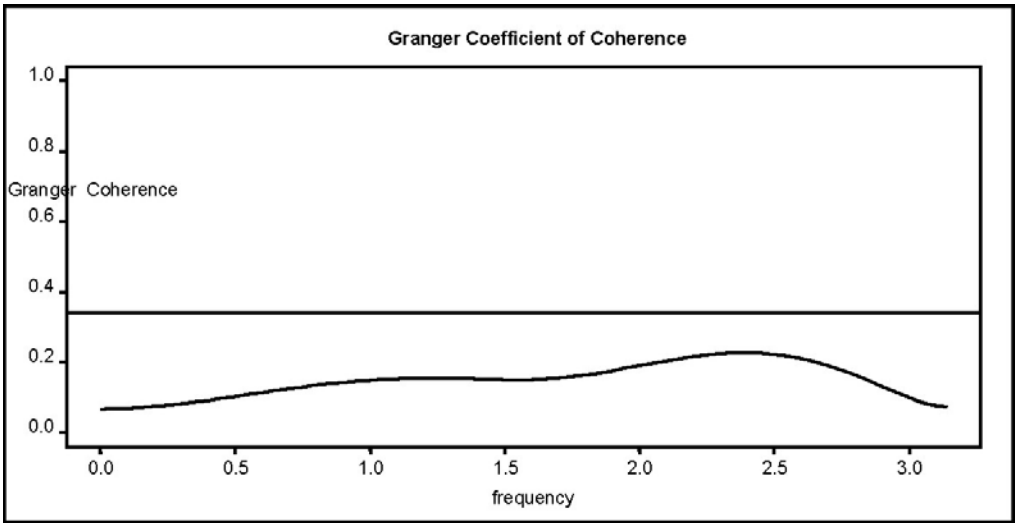


Figure 6

Frequency Domain GC running from Energy Consumption to Output.



IV. Conclusions

This paper examines the causal relationship between energy consumption and economic growth in three African countries (i.e., Algeria, Egypt, and South Africa) from 1970 to 2011 using the frequency domain-based GC test proposed by Lemmens *et al*.

(2008). Our empirical results support a unidirectional causality running from energy consumption to economic growth for Algeria, a bi-directional causality between energy consumption and economic growth for Egypt, and no causality in any direction between energy consumption and economic growth for South Africa. Our findings provide important policy implications for energy policies and strategies in the African countries studied.

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