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# INVESTOR SENTIMENT, EXTRAPOLATION AND ASSET PRICING

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## Abstract

*We develop an asset pricing model with investor sentiment and extrapolative behavior by assuming that there are many investors in the market who form their stock demand by weighting the sentiment signal, the extrapolative signal and the value signal. Our model predicts that both investor sentiment and extrapolation impose positive effects on the stock price deviation from fundamental value, and the direction and magnitude of stock price deviation depend on the relative strength of the sentiment signal and the extrapolation signal. Futhermore, we find that the weights of the sentiment signal and the extrapolative signal are positively related to the short-term correlation of stock returns, while the lagged weight of sentiment signal negatively effects the short-term correlation of stock returns. Moreover, the model also predicts that the sentiment signal and extrapolative signal weights are positively correlated with stock volatility, and extrapolative behavior exacerbates the sentiment-driven stock volatility due to extrapolating endogenous stock returns. Finally, we find empirical evidence consistent with the model's predictions.*

**Keywords:** investor sentiment; extrapolation; price deviation; return correlation; excess volatility
**JEL Classification:** G12; G14

## 1. Introduction

The important role played by investor sentiment in financial markets has been confirmed in the rapidly expanding literature (De Long *et al*. 1990; Baker and Wurgler 2006, 2007; Stambaugh *et al.*, 2012, 2014; Han and Li, 2017; Karavias *et al*., 2021). An alternative theory holds that extrapolative expectation, the idea that investors' beliefs about an asset's future return are a positive function of the asset's recent past returns, can explain a wide set of stylized facts in asset prices (Greenwood and Shleifer, 2014; Glaeser and Nathanson, 2017; Barberis et al., 2018; Da *et al.*, 2021; Jin and Sui, 2022; Liao *et al*., 2022). In addition, some related literature attributes investor sentiment to extrapolative behavior in the financial

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market (Barberis *et al*., 2015;Cassella and Gulen, 2019; Atmaz, 2022). In fact, extrapolation and investor sentiment have two different bases for investors to form expected returns: extrapolation - investors' formation of expected returns based on past returns; investor sentiment – the formation of expected return by investors based on current investor enthusiasm[3]. However, existing research generally examines the response of asset prices to either investor sentiment or extrapolative behavior. Few studies have explored the effects of both investor sentiment and extrapolative belief on asset prices, especially when investor sentiment and extrapolation point in the opposite direction. Therefore, this paper extends current research by studying the combined effects of investor sentiment and extrapolative behavior on asset pricing.

In this paper, we present an equilibrium model based on investor sentiment and extrapolative behavior to formalize their combined effects on the asset pricing. In the model, the economy contains two assets: a risk-free bond with a fixed return, and a risky stock that pays a liquidating dividend at a fixed time in the future. The stock price is determined in equilibrium. There are two types of investors, and both types of investors maximize the expected utility of wealth in the next period. The first type is sophisticated investors. In particular, we assume that sophisticated investors form their stock demand based on three signals: a sentiment signal which depends on investor's current sentiment, an extrapolative signal that responds to past change in the stock price, and a value signal that measures the difference between the stock price and the fundamental value of the stock. Such a hypothesis has a biological foundation, Fehr and Rangel (2011) and Towal *et al*. (2013) argue that individual decisions are shaped by the allocation of attention to various attributes of the choice. On the one hand, the sophisticated investors believe that both investor sentiment and extrapolation are priced drivers in market equilibrium; therefore, if the stock price has recently risen and current investor sentiment is optimistic, sophisticated investors would increase their demand for the stock. On the other hand, if the stock price deviates far from the fundamental value, the sophisticated investors rationally recognize the dangers that the stock price falling to the fundamental value. As a result, to capture the idea that all three signals can influence sophisticated investors' decision, we assume that sophisticated investors' demand is a weighted average of the three signals. The second type of investors are fundamental investors who form their demand for the stock based on the value signal alone, buying the stock when its price is low relative to the fundamental value and selling when its price is high.

First, we use our model to understand how investor sentiment and extrapolation influence stock price stability. We find that the direction and the magnitude of stock price deviation from fundamental value depends on the relative strength of sentiment signal and the extrapolative signal. When the sentiment signal and the extrapolative signal point in the same direction, the significant sentiment signal would amplify the price error, which could be called "superimposed effect" of investor sentiment; when sentiment signal and extrapolative

[3] *Although some scholars have argued that investor sentiment may relate to extrapolation, they are distinct concepts. Generally, investor sentiment captures the prevailing level of optimism or pessimism about risky asset (Baker and Wulger,2006), and is a summary of many potential channels including anchoring, overconfidence, and so on. In this sense, extrapolation can be viewed as one specific psychological channel for investor sentiment. Thus, investor sentiment does not completely depend on extrapolative belief (He et al.,2020). For example, following positive stock returns, investor sentiment could be either rise or fall and becomes more bullish or bearish.*

signal point in opposite direction, significant sentiment signal would change the direction of pricing error, which might be called “inverse effect” of investor sentiment. Thus, compared to De Long *et al*. (1990) and Barberis *et al.* (2018), due to the interaction of investor sentiment and extrapolation, there are several possible stock price paths.

Next, we examine how investor sentiment and extrapolative belief influence stock return correlation. Our model predicts the short-term correlation of stock returns increases with the weight of the sentiment signal, while it decreases with the lagged weight of the sentiment signal. Moreover, extrapolation also leads to a short-term momentum effect, and the positive correlation of short-term stock returns increases with the weight of the extrapolative signal. We further investigate the combined effect of investor sentiment and extrapolation on the correlation of stock returns and find that when the lagged weight of the sentiment signal is sufficiently large, short-term stock returns would exhibit a reversal effect; otherwise, short-term stock returns would exhibit a momentum effect.

Finally, we investigate the combined effect of investor sentiment and extrapolation on stock return volatility. Consistent with Barberis et al. (2015), Atmaz (2022) and others, the model predicts that extrapolative behavior leads to excess stock volatility, and the higher the weight of the extrapolative signal, the higher the stock volatility. The model also finds that investor sentiment can also lead to excessive stock price volatility, and stock volatility increases with the weight of the sentiment signal. More importantly, we show that extrapolation can exacerbate sentiment-driven stock volatility due to the extrapolation of endogenous stock returns.

We find evidence consistent with model predictions. First, we examine the combined effect of investor sentiment and extrapolation on stock price deviation. We uncover that the stock price is overvalued among stocks with positive investor sentiment and higher extrapolative belief but undervalued among stocks with negative investor sentiment and lower extrapolative belief, and both investor sentiment and extrapolation positively effect stock price deviation. Second, we capture the individual stock loadings on individual investor sentiment and extrapolation by sentiment beta and extrapolation beta, respectively. We show that the correlation of the short-term stock returns increases with both sentiment beta and extrapolation beta, while decreases with lagged sentiment beta. Lastly, we also show that stock return volatility increases with both sentiment beta and extrapolation beta, and extrapolation beta can positively influence the sentiment-driven stock return volatility.

This paper contributes to the existing literature includes the following ways. First, we show that the direction and the magnitude of pricing error depend on the relative strength of the sentiment signal and the extrapolative signal. Second, we find that despite the weight of the sentiment signal and the extrapolative signal positively influence short-term correlation of stock returns, the lagged weight of sentiment signal positively predicts short-term correlation of stock returns. Therefore, whether the short-term stock returns show a momentum effect or a reverse effect depends on the change in the weight of the sentiment signal. Third, both sentiment and extrapolative signal weights positively influence stock volatility, which is consistent with existing studies related to investor sentiment and extrapolation. Notably, due to extrapolating endogenous stock returns, extrapolative behavior exacerbates the stock volatility driven by investor sentiment.

The balance of the paper is organized as follows. We formally present and solve the model in Section 2. Section 3 provides some basic analysis of equilibrium. Section 4 conducts empirical analysis, and Section 5 concludes.

## 2. The Model

There are two assets in the economy: a risk-free bond and a risky stock. The bond is in perfectly elastic supply and pays a risk-free interest rate which is normalized to zero. The stock has a fixed supply $Q$. There are $T+1$ dates, $t = 0, 1, \cdots, T$. Trading begins at date 0 and ends at date $T$. The stock pays a single liquidating dividend at the terminal date $T$. The ultimate value of the liquidating dividend can be written as

$$D_T = D_0 + \varepsilon_1 + \varepsilon_2 + \cdots + \varepsilon_T , \tag{1}$$

where all the $\varepsilon$'s are independently distributed, mean-zero normal random variables with variance $\sigma_\varepsilon^2$. The process $\varepsilon_t$ represents the arrival news about $D_T$. The price of stock price, $p_t$, is determined endogenously. There are two types of investors in the economy: fundamental investors and sophisticated investors. Fundamental investors, whose demand at all times depends only on the price of stock relative to its fundamental value, are denoted $F$ and make up a fraction $\mu^F$ of the population. Sophisticated investors, who are denoted $S$, form their demand by weighing three signals: extrapolative signal, sentiment signal and value signal. All investors have constant absolute risk aversion (*CARA*) utility with the same risk aversion coefficient of $\gamma$ ($\gamma > 0$).

### 2.1. The demand of fundamental investors

We assume that fundamental investors are bounded rationality and expect the stock price to revert to fundamental value within one period. According to Barberis et al. (2018), the stock demand of fundamental investors at time $t$, denoted by $N_t^F$,satisfies the following equation

$$N_t^F = \frac{D_t - \gamma\sigma_\varepsilon^2 (T-t-1)Q - p_t}{\gamma\sigma_\varepsilon^2} . \tag{2}$$

In particular, if all investors are the fundamental investors (i.e., $\mu^F = 1$), then, setting Eq. (2) equals to the stock supply $Q$, the equilibrium stock price can be written as

$$p_t^F = D_t - \gamma\sigma_\varepsilon^2 (T-t)Q , \tag{3}$$

which we regard as the fundamental value of the stock.

### 2.2. The demand of sophisticated investors

In our paper, in addition to how the stock price compares to its fundamental value, the sophisticated investors also pay attention to current investor sentiment and the past trend of the stock price. We refer to the three components that sophisticated investors pay attention to as “value signal”, “sentiment signal” and “extrapolative signal”, respectively. And Eq. (2) can be viewed as “value signal”. In what follows, we will specify “sentiment signal” and “extrapolative signal”, respectively.

#### 2.2.1. Sentiment signal

Consider an economy with timing and asset structure described at the start of Section 2. Now consider an investor who is vulnerable to sentiment shocks and perceives the stock return with investor sentiment. Let us assume that investor sentiment follows a random walk process, $S_t = S_{t-1} + \eta_t$, where all the $\eta_t$'s are independently distributed, mean-zero normal random variables with variance $\sigma_\eta^2$. The correlation coefficient between $\varepsilon_t$ and $\eta_t$ is $\rho_t$.[4] At date $t$, the investor chooses his stock demand, denoted by $N_t^N$, by maximizing expected next period's wealth utility. Since the next period's wealth is

$$W_{t+1} = W_t + N_t^N \left(p_{t+1} - p_t\right), \tag{4}$$

then his objective is

$$\max_{N_t^N} \; E_t\left(-\exp(-\gamma W_{t+1}\right). \tag{5}$$

The first-order condition gives the optimal demand is (see the Appendix A in the Online Appendix)

$$N_t^N = \frac{E_t^N\left(p_{t+1} - p_t\right)}{\gamma Var_t^N\left(p_{t+1} - p_t\right)}, \tag{6}$$

where, for simplicity, we assume that the investor takes the conditional distribution of the future price change to be normal. We further assume that the investor believes that future price change depends on future investor sentiment, *i.e.*, $p_{t+1} - p_t = F\left(S_{t+1}\right)$, where the sentiment function $F\left(S_{t+1}\right)$ is a monotonous increasing function of investor sentiment. For simplicity, we set $F(S_{t+1}) = S_{t+1}$ .Thus, we can obtain $E_t^N\left(p_{t+1} - p_t\right) = S_t$ , and $Var_t^N\left(p_{t+1} - p_t\right) = \sigma_\eta^2$. Consequently, the demand of the investor at date $t$ becomes

$$N_t^N = \frac{S_t}{\gamma \sigma_\eta^2}, \tag{7}$$

which can be viewed as "sentiment signal".

### 2.2.2 Extrapolative signal

Consider an economy with timing and asset structure described at the start of Section 2. Now consider an investor who forms beliefs about the future stock price change by extrapolating the stock's past price changes (Hong and Stein, 1999; Barberis *et al*., 2015, 2018; Da *et al*., 2021; Jin and Sui,2022). At date $t$, the investor chooses his stock demand, which is denoted by $N_t^E$, by maximizing expected next period's wealth utility. The next period's wealth is

$$W_{t+1} = W_t + N_t^E \left(p_{t+1} - p_t\right), \tag{8}$$

then his objective is

$$\max_{N_t^E} \; E_t\left(-\exp(-\gamma W_{t+1}\right). \tag{9}$$

The first-order condition gives his optimal demand

$$N_t^E = \frac{E_t^E\left(p_{t+1} - p_t\right)}{\gamma Var_t^E\left(p_{t+1} - p_t\right)}. \tag{10}$$

[4] *In our analysis, we impose the restriction $0 \le \rho_t < 1$ to restrict our attention to the case of positive correlation between the cash-flow news and the changes in investor sentiment. This assumption is intuitive since when cash-flow news is good (bad), the view on the stock becomes relatively more optimistic (pessimistic).*

where, we assume that the investor takes the conditional distribution of the future price change to be normal. We further assume that the investor makes forecasts the future price change based on the cumulative price changes over the past $k$ periods, namely, $p_{t-1} - p_{t-k-1}$. Hong and Stein (1999) has turned out that the exact value of $k$ is not important, thus, for simplicity, we set $k = 1$, which implies that $E_t^E\left(p_{t+1} - p_t\right) = \beta\left(p_{t-1} - p_{t-2}\right)$, where $\beta$ is positive feedback coefficient. Following Barberis *et al*. (2018), we further assume that the investor sets the conditional variance of the future price change equal to the variance of cash-flow shocks (*i.e.*, $Var_t^E\left(p_{t+1} - p_t\right) = \sigma_\varepsilon^2$). Thus, the demand of the investor at date $t$ can be given as

$$N_t^E = \frac{\beta\left(p_{t-1} - p_{t-2}\right)}{\gamma\sigma_\varepsilon^2}, \tag{11}$$

which can be viewed as “extrapolative signal”.

The demand of sophisticated investors, denoted by $N_t^S$, is a weighted average of value signal, sentiment signal and extrapolative signal, thus, $N_t^S$ can be given by

$$N_t^S = w_{1,t}\left(\frac{D_t - \gamma\sigma_\varepsilon^2\left(T - t - 1\right)Q - p_t}{\gamma\sigma_\varepsilon^2}\right) + w_{2,t}\frac{S_t}{\gamma\sigma_\eta^2} + w_{3,t}\frac{\beta\left(p_{t-1} - p_{t-2}\right)}{\gamma\sigma_\varepsilon^2}, \tag{12}$$

Where $w_{1,t}$, $w_{2,t}$ and $w_{3,t}$ are the weights of sophisticated investors on value signal, sentiment signal and extrapolative signal at date $t$, respectively, and satisfy $w_{1,t}+w_{2,t}+w_{3,t}=1$.

### 2.3 The equilibrium of the economy

With all this in hand, we proceed to solve the model. Imposing market clearing condition can give

$$\mu^F N_t^F + \left(1 - \mu^F\right)N_t^S = Q. \tag{13}$$

Solving for the equilibrium stock price gives

$$\begin{aligned} p_t = {} & D_t + \frac{\left(1 - \mu^F\right)w_{2,t}}{\mu^F + \left(1 - \mu^F\right)w_{1,t}}\frac{\sigma_\varepsilon^2}{\sigma_\eta^2}S_t + \frac{\left(1 - \mu^F\right)w_{3,t}}{\mu^F + \left(1 - \mu^F\right)w_{1,t}}\beta\left(p_{t-1} - p_{t-2}\right) \\ & - \left(T - t - 1 + \frac{1}{\mu^F + \left(1 - \mu^F\right)w_{1,t}}\right)Q\gamma\sigma_\varepsilon^2, \end{aligned} \tag{14}$$

where the sentiment elasticity and the extrapolation elasticity of the stock price are, $\frac{\left(1 - \mu^F\right)w_{2,t}}{\mu^F + \left(1 - \mu^F\right)w_{1,t}}\frac{\sigma_\varepsilon^2}{\sigma_\eta^2}$ and $\frac{\left(1 - \mu^F\right)w_{3,t}}{\mu^F + \left(1 - \mu^F\right)w_{1,t}}$, respectively. Consequently, in the economy with investor sentiment and extrapolative beliefs, the stock price is increasing in investor sentiment（$S_t$）and extrapolative belief ($\beta\left(p_{t-1} - p_{t-2}\right)$); the investor sentiment elasticity is increasing in the weight of sophisticated investors on sentiment signal（$w_{2,t}$）; and the extrapolation elasticity is increasing in the weight of sophisticated investors on extrapolative signal（$w_{3,t}$）.

To further understand the role that investor sentiment and extrapolation play in our model, we compare the model's predictions to an economy where all investors are fundamental investors, or sophisticated investors either do not pay attention to the sentiment signal or the extrapolative signal. If sophisticated investors do not pay attention to the extrapolative signal, the equilibrium stock price is

$$p_t^* = D_t + \frac{\left(1-\mu^F\right)w_{2,t}^*}{\mu^F + \left(1-\mu^F\right)w_{1,t}^*}\frac{\sigma_\varepsilon^2}{\sigma_\eta^2}S_t - \left(T-t-1+\frac{1}{\mu^F+\left(1-\mu^F\right)w_{1,t}^*}\right)Q\gamma\sigma_\varepsilon^2, \tag{15}$$

where $w_{1,t}^*$ and $w_{2,t}^*$ are the weights of sophisticated investors on the value signal and sentiment signal, respectively, at date $t$ and satisfy $w_{1,t}^*+w_{2,t}^*=1$.

If sophisticated investors do not pay attention to sentiment signal, the equilibrium stock price is

$$p_t^{**} = D_t + \frac{\left(1-\mu^F\right)w_{3,t}^{**}}{\mu^F + \left(1-\mu^F\right)w_{1,t}^{**}}\beta\left(p_{t-1}^{**} - p_{t-2}^{**}\right) - \left(T-t-1+\frac{1}{\mu^F+\left(1-\mu^F\right)w_{1,t}^{**}}\right)Q\gamma\sigma_\varepsilon^2, \tag{16}$$

where $w_{1,t}^{**}$ and $w_{3,t}^{**}$ are the weights of sophisticated investors on value signal and extrapolative signal, at date $t$, respectively, and satisfy $w_{1,t}^{**}+w_{3,t}^{**}=1$.

# 3. Stock Price Deviation, Correlation of Stock Returns, and Stock Return Volatility

In this section, we illustrate the properties of the equilibrium in more detail. Specifically, we analyze the combined effects of investor sentiment and extrapolation on the stock price stability, the correlation between stock returns and the stock price volatility.

### *3.1 The Deviation of Stock Price from the Fundamental Value*

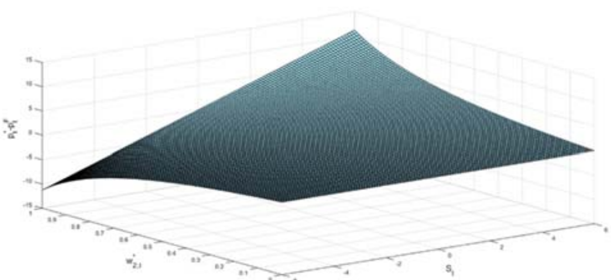
The deviation of stock price from fundamental value can be written as $p_t - p_t^F$, if $p_t - p_t^F > 0$ ($p_t - p_t^F < 0$), then the stock price is overvalued (undervalued). To quantitatively illustrate the impact of investor sentiment and extrapolative belief on stock price stability, we give a 3-dates trading equilibrium numerical simulation and the parameters are chosen as follows: $D_0 = 10$, $\sigma_\varepsilon{}^2 = 0.2$, $\sigma_\eta{}^2 = 0.25$, $Q = 1$, $\gamma = 0.1$, $\mu^F = 0.3$, $\beta$=0.5, $\rho_1$=0.1, $S_t \in [-6,6]$ for $t = 0,1,2$, and a particular set of values of cash-flow shocks is $\varepsilon_1$=2, $\varepsilon_2$=0 and $\varepsilon_3$=0.

We first consider the case that the sophisticated investors pay attention to value signal and sentiment signal, and investigate how investor sentiment impact stock price stability. From Eq. (3) and Eq. (15), the deviation of stock price from fundamental value can be written as

$$p_t^{*} - p_t^F = \frac{\left(1-\mu^F\right) w_{2,t}^{*}}{\mu^F + \left(1-\mu^F\right) w_{1,t}^{*}} \frac{\sigma_\varepsilon^{\ 2}}{\sigma_\eta^{\ 2}} S_t + \left(1 - \frac{1}{\mu^F + \left(1-\mu^F\right) w_{1,t}^{*}}\right) \gamma \sigma_\varepsilon^2 Q \ , \tag{17}$$

where $w_{1,t}^{*}+w_{2,t}^{*}=1$, for $t=0,1,2$. Eq. (17) demonstrates that the stock price deviation depends on the intensity of investor sentiment and sophisticated investors' weight on sentiment signal. In particular, Figure 1 shows that if investor sentiment is optimistic, then the more optimistic（pessimistic）investor sentiment is and the stronger the weight of sophisticated investors on the sentiment signal, the greater the degree of overvaluation（undervaluation）.

**Figure 1. The deviation of stock price from the fundamental value caused by investor sentiment**



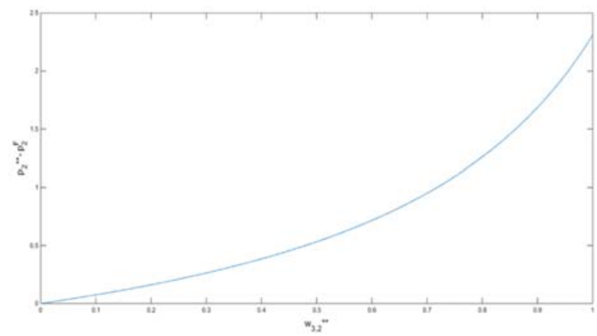
*Notes: The figure illustrates the sensitivity of the stock price deviation ($p_t^{*} - p_t^F$) to investor sentiment ($S_t$) and sophisticated investors' weight on sentiment signal ($w_{2,t}^{*}$) at date $t$. The parameters are chosen as follows: $\sigma_\varepsilon^{\ 2} = 0.2, \sigma_\eta^{\ 2} = 0.25, Q = 1, \gamma = 0.1, \mu^F = 0.3$.*

We then consider the case that the sophisticated investors pay attention to the value signal and the extrapolative signal and investigate how extrapolation affects stock price stability. Since the sophisticated investors do not pay attention to the sentiment signal, then $p_0^{**}=p_0^F$ and $p_1^{**}=p_1^F$. The deviation of stock price from fundamental value at date 2 can be written as

$$p_2^{**} - p_2^F = \frac{\left(1-\mu^F\right) w_{3,2}^{**}}{\mu^F + \left(1-\mu^F\right) w_{1,2}^{**}} \beta\left(p_1^{**} - p_0^{**}\right) + \left(1 - \frac{1}{\mu^F + \left(1-\mu^F\right) w_{1,2}^{**}}\right) \gamma \sigma_\varepsilon^2 Q \ , \tag{18}$$

where $w_{1,2}^{**}+w_{3,2}^{**}=1$. Therefore, the positive cash-flow shock at date 1 pushes the stock price at date 1, $p_1^{**}$, up, which causes sophisticated investors to sharply increase their demand for the stock at date 2 and in turn pushes up the stock price at date 2 well above fundamental value. Figure 2 quantitatively illustrates the relationship between $p_2^{**} - p_2^F$ and $w_{3,2}^{**}$ and shows that the stronger the weight of sophisticated investors on the extrapolative signal, the greater the degree of overvaluation.

**Figure 2. The deviation of stock price from the fundamental value caused by extrapolation**



*Notes. The figure illustrates the sensitivity of the deviation of stock price from fundamental value at date 2 ($p_2^{**} - p_2^F$) to sophisticated investors' weight on extrapolative signal at date 2 ( $w_{3,2}^{**}$ ). The parameters are chosen as follows: $\sigma_\varepsilon^2 = 0.2, \sigma_\eta^2 = 0.25, Q = 1, \gamma = 0.1, \mu^F = 0.3$, and $\varepsilon_1$=2.*

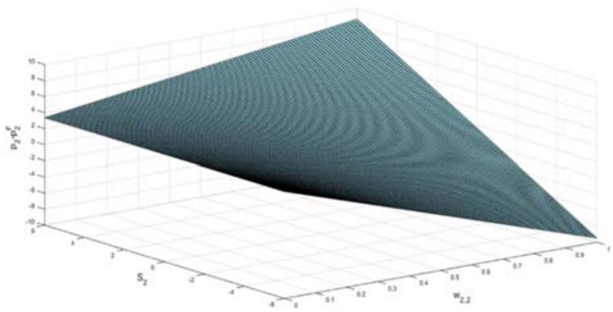
Finally, we consider the situation where sophisticated investors pay attention to both the sentiment signal and the extrapolative signal and investigate the combined effect of investor sentiment and extrapolation on stock price stability. At date 2, the deviation of the stock price from the fundamental value can be given by

$$p_2 - p_2^F = \frac{\left(1-\mu^F\right)w_{2,2}}{\mu^F + \left(1-\mu^F\right)w_{1,2}} \frac{\sigma_\varepsilon^2}{\sigma_\eta^2} S_2 + \frac{\left(1-\mu^F\right)w_{3,2}}{\mu^F + \left(1-\mu^F\right)w_{1,2}} \beta\left(p_1 - p_0\right) + \left(1 - \frac{1}{\mu^F + \left(1-\mu^F\right)w_{1,2}}\right) Q\gamma\sigma_\varepsilon^2 \quad (19)$$

Eq. (19) shows that both investor sentiment and extrapolative belief positively influence stock price deviation. Although extrapolation could be one of many driving forces behind overall investor sentiment (He *et.al*., 2020), the direction of investor sentiment and extrapolation can be opposite. That is, following positive (negative) stock returns, investor sentiment may become more bearish (bullish) due to other forces such as anchoring bias or overconfidence. In these circumstances, the magnitude and direction of the stock price deviation is determined by the relative strength of the sentiment signal and the extrapolative signal. To quantitatively illustrate the combined effect of investor sentiment and extrapolation on stock price deviation, we give a numerical example in Figure 3 and assume that the stock price has increased, and further set $\beta\left(p_1 - p_0\right) = 2$ and $w_{1,2}$=0.1.

Figure 3 indicates that when investor sentiment is optimistic, if the sentiment signal is more (less) significant than extrapolative signal, $\frac{S_2}{\gamma\sigma_\eta^2} > \frac{\beta\left(p_1 - p_0\right)}{\gamma\sigma_\varepsilon^2} > 0$ ( $0 < \frac{S_2}{\gamma\sigma_\eta^2} < \frac{\beta\left(p_1 - p_0\right)}{\gamma\sigma_\varepsilon^2}$ ), then the stronger the weight of sophisticated investors on sentiment signal, the larger (lower) degree of overvaluation.

**Figure 3. The deviation of stock price from the fundamental value caused by investor sentiment and extrapolation**

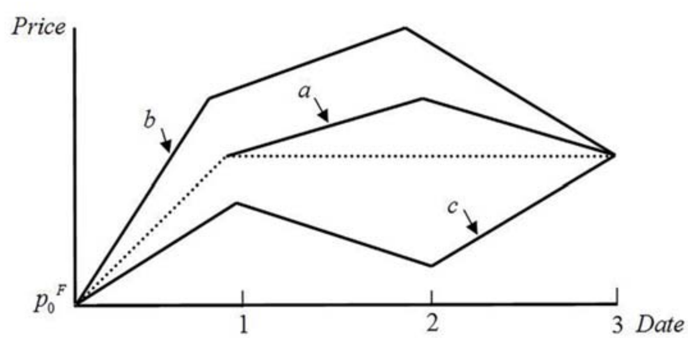


*Note. The figure illustrates the sensitivity of stock price deviation ($p_2 - p_2^F$) to investor sentiment ($S_2$) and the weight of sentiment signal ($w_{2,2}$). The parameters are chosen as follows: $\sigma_\varepsilon{}^2 = 0.2, \sigma_\eta{}^2 = 0.25, Q = 1, \gamma = 0.1, \mu^F = 0.3, \beta(p_1 - p_0) = 2$ and $w_{1,2} = 0.1$.*

When investor sentiment is pessimistic, if the strength of the sentiment signal is lower than the extrapolative signal, $\left|\frac{w_{2,2}S_2}{\gamma\sigma_\eta^2}\right| < \frac{w_{3,2}\beta(p_1 - p_0)}{\gamma\sigma_\varepsilon^2}$, then the stock price is still overvalued; if the strength of sentiment signal is greater than the extrapolative signal, $\left|\frac{w_{2,2}S_2}{\gamma\sigma_\eta^2}\right| > \frac{w_{3,2}\beta(p_1 - p_0)}{\gamma\sigma_\varepsilon^2}$, then the stock price would be undervalued.

Given the particular 3-date sequence of cash-flow shocks, $\varepsilon_1$=2 , $\varepsilon_2$=0 and $\varepsilon_3$=0 , Figure 4 shows the possible stock price paths as a function of time for the 3-date trading equilibrium.

**Figure 4. The possible stock price paths**




*Note: The figure illustrates the possible stock price paths with a particular set of values of cash-flow shocks $\varepsilon_1$=2, $\varepsilon_2$=0 and $\varepsilon_3$=0.*

The middle dashed curve in Figure 4 exhibits the fundamental value path conditional on date 3 correction. The curve $a$ in Figure 4 illustrates the stock price path when the pricing error is caused by extrapolation. At date 0 and date 1, the stock prices equal to the fundamental values. Since there is a positive cash-flow shock release at date 1, then both fundamental investors and sophisticated investors buy the stock based on positive fundamental information, which increases the stock price at date 1. At date 2, sophisticated investors continue to buy the stock based on the positive extrapolative signal leading to overvaluation of the stock price, and the degree of overvaluation depends on the weight of sophisticated investors on the extrapolative signal. The curve $b$ and $c$ illustrate the case where sophisticated investors pay some attention to sentiment signal, and for simplicity we assume $S_0 = 0$. The curve $b$ illustrates the stock price path when the sentiment signal and the extrapolative signal point in the same direction, and the strength of sentiment signal is greater than extrapolative signal, then investor sentiment amplifies the price error, which might be called "superimposed effect" of investor sentiment. At date 1, investor sentiment is optimistic, then sophisticated investors demand is high, exerting upward pressure on stock price which leads to stock price overreaction; at date 2, positive extrapolative signal and significant optimistic sentiment signal lead to stock price continuing overreaction and the degree of overvaluation is higher than curve $a$. The curve $c$ illustrates the stock price path when sentiment signal and extrapolative signal point in opposite direction, and the strength of sentiment signal is greater than extrapolative signal, then pessimistic investor sentiment would lead to stock price undervaluation, which might be called "inverse effect" of investor sentiment. At date 1, the pessimistic investor sentiment leads to stock price underreaction; at date 2, although the extrapolative signal may be positive, significant pessimistic sentiment signal leads to stock price continuing undervaluation and the stock price may be lower than the stock price at date 1. The curve $c$ exhibits a special phenomenon of upward price spiral.

### 3.2 The correlation of stock returns

We now examine what our model predicts about the correlation structure of stock returns and try to provide a new perspective to understand the pattern of the stock returns correlation. To gain better insight into the correlation between stock returns, we evaluate the comparative statistics of the correlation of stock returns in the cases where there are only fundamental investors in the economy, sophisticated investors do not pay attention to the sentiment signal, or sophisticated investors do not pay attention to extrapolative signal. First, suppose that all investors in the economy are fundamental investors (i.e., $\mu^F = 1$), the covariance of stock returns can be written as $Cov\left(p_1^F - p_0^F, p_2^F - p_1^F\right)=0$ and $Cov\left(p_1^F - p_0^F, p_3^F - p_2^F\right)=0$, which imply that the stock returns are uncorrelated and unpredictable.

Second, we investigate how investor sentiment influences the correlation between stock returns. From Eq. (15), the covariance of stock returns becomes

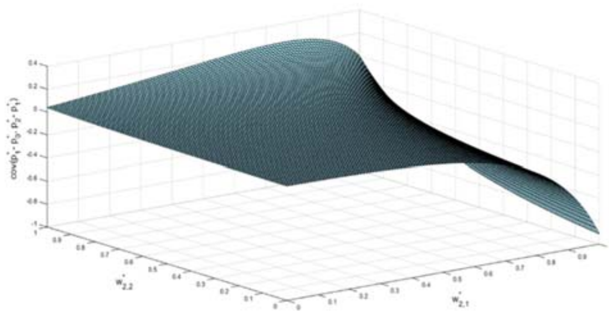
$$Cov(p_1^* - p_0^*, p_2^* - p_1^*) = \left( \frac{\left(1-\mu^F\right)w_{2,2}^*}{\mu^F + \left(1-\mu^F\right)w_{1,2}^*} - \frac{\left(1-\mu^F\right)w_{2,1}^*}{\mu^F + \left(1-\mu^F\right)w_{1,1}^*} \right) \times \left( \frac{\left(1-\mu^F\right)w_{2,1}^*}{\mu^F + \left(1-\mu^F\right)w_{1,1}^*} \left( \frac{\sigma_\varepsilon^2}{\sigma_\eta^2} \right) \sigma_\varepsilon^2 + \rho_1 \left( \frac{\sigma_\varepsilon}{\sigma_\eta} \right) \sigma_\varepsilon^2 \right), \tag{20}$$

and

$$Cov\left(p_1^* - p_0^*, p_3^* - p_2^*\right) = -\frac{\left(1-\mu^F\right)w_{2,2}^*}{\mu^F + \left(1-\mu^F\right)w_{1,2}^*} \left( \frac{\left(1-\mu^F\right)w_{2,1}^*}{\mu^F + \left(1-\mu^F\right)w_{1,1}^*} \left( \frac{\sigma_\varepsilon^2}{\sigma_\eta^2} \right) \sigma_\varepsilon^2 + \rho_1 \left( \frac{\sigma_\varepsilon}{\sigma_\eta} \right) \sigma_\varepsilon^2 \right). \tag{21}$$

As a novel result, we find that the short-term correlation of the stock returns increases with the weight of sentiment signal at date 2 ( $w_{2,2}^*$ ), but decreases with the weight of sentiment signal at date1 ( $w_{2,1}^*$ ). Consequently, whether the short-term stock returns exhibit momentum effect or reverse effect depends on the relative weight of sentiment signal at date 2 and date 1. In particular, Eq. (20) shows that if the weight of sentiment signal at date 2 is larger (smaller) than the weight at date 1, $w_{2,2}^* > w_{2,1}^*$ ( $w_{2,2}^* < w_{2,1}^*$ ), then investor sentiment leads to short-term momentum (reversal) effect, $Cov(p_1^* - p_0^*, p_2^* - p_1^*) > 0$ ( $Cov(p_1^* - p_0^*, p_2^* - p_1^*) < 0$ ). We further find that the magnitude of short-term momentum effect or short-term reversal effect depends not only on $\left| \frac{\left(1-\mu^F\right)w_{2,2}^*}{\mu^F + \left(1-\mu^F\right)w_{1,2}^*} - \frac{\left(1-\mu^F\right)w_{2,1}^*}{\mu^F + \left(1-\mu^F\right)w_{1,1}^*} \right|$ but also on $\frac{\left(1-\mu^F\right)w_{2,1}^*}{\mu^F + \left(1-\mu^F\right)w_{1,1}^*}$ . Then we give a numerical example in Figure 5, and the parameters are chosen as those of Figure 1. Figure 5 shows that, given the weight of sophisticated investors on sentiment signal at date 1, increasing the weight of the sentiment signal at date 2 would increase the momentum effect or reduce the reversal effect. Now ask what happens when the weight of sophisticated investors on sentiment signal at date 1, $w_{2,1}^*$ , is varied. As we can see from Figure 5, the covariance of the stock returns is nonmonotonic. For example, given $w_{2,2}^* = 0.7$ , with $w_{2,1}^* = 0.2$ , the covariance of the stock returns is 0.03435, and with $w_{2,1}^* = 0.85$ ,the covariance of the stock returns is -0.1281, not as large when $w_{2,1}^* = 0.5$ and the covariance of the stock returns is 0.4356. Furthermore, when $w_{2,1}^* = 0.75$ and $w_{2,2}^* = 1$ , the momentum effect is largest; and when $w_{2,1}^* = 1$ and $w_{2,2}^* = 0$ , the reversal effect is largest.

**Figure 5. The short-term covariance of stock returns caused by investor sentiment**



*Note: The figure illustrates the sensitivity of short-term covariance of stock returns ($Cov(p_1{}^* - p_0{}^*, p_2{}^* - p_1{}^*)$) to sophisticated investors' weight on sentiment signal at date 1 ($w_{2,1}{}^*$) and date 2 ($w_{2,2}{}^*$). The parameters are chosen as follows:$\sigma_\varepsilon{}^2 = 0.2$,$\sigma_\eta{}^2 = 0.25$,$Q = 1$,$\gamma = 0.1$,$\mu^F = 0.3$ and $\rho_1 = 0.1$.*

Furthermore, Eq. (21) shows that investor sentiment can always lead to a long-term reversal effect and the higher the weight of the sentiment signal, the greater the long-term reversal effect.

Now, we discuss the impact of extrapolation on the correlation of stock returns. From Eq.(16), the covariance of stock returns can be given by

$$Cov(p_1^{**} - p_0^{**}, p_2^{**} - p_1^{**}) = \frac{\left(1-\mu^F\right) w_{3,2}{}^{**} \beta}{\mu^F + \left(1-\mu^F\right) w_{1,2}{}^{**}} \sigma_\varepsilon^2 , \tag{22}$$

and

$$Cov\left(p_1^{**} - p_0^{**}, p_3^{**} - p_2^{**}\right) = -\frac{\left(1-\mu^F\right) w_{3,2}{}^{**} \beta}{\mu^F + \left(1-\mu^F\right) w_{1,2}{}^{**}} \sigma_\varepsilon^2 \tag{23}$$

Eqs (22) and (23) show that extrapolation leads to short-term momentum effect and long-term reversal effect. Furthermore, the stronger the weight of sophisticated investors on extrapolative signal, the greater the momentum effect and reversal effect.

Each of the three special cases above may have been studied separately, and finally we look at what happens when sophisticated investors pay attention to each signal. From Eq. (14), the covariance of stock returns can be given by (see the Appendix B in the Online Appendix)

$$Cov(p_1 - p_0, p_2 - p_1) = a\sigma_\varepsilon^2 + b\left(\frac{\sigma_\varepsilon^2}{\sigma_\eta^2}\right)\sigma_\varepsilon^2 + c\rho_1\left(\frac{\sigma_\varepsilon}{\sigma_\eta}\right)\sigma_\varepsilon^2 , \tag{24}$$

and

$$Cov\left(p_1 - p_0, p_3 - p_2\right) = -a\sigma_\varepsilon^2 - d\left(\frac{\sigma_\varepsilon^2}{\sigma_\eta^2}\right)\sigma_\varepsilon^2 - e\rho_1\left(\frac{\sigma_\varepsilon}{\sigma_\eta}\right)\sigma_\varepsilon^2 , \tag{25}$$

where

$$a=\frac{\left(1-\mu^F\right)w_{3,2}}{\mu^F+\left(1-\mu^F\right)w_{1,2}}\beta\,, \tag{26}$$

$$b=\frac{\left(1-\mu^F\right)w_{2,1}}{\mu^F+\left(1-\mu^F\right)w_{1,1}}\left(\begin{array}{l}\dfrac{\left(1-\mu^F\right)w_{2,2}}{\mu^F+\left(1-\mu^F\right)w_{1,2}}-\dfrac{\left(1-\mu^F\right)w_{2,1}}{\mu^F+\left(1-\mu^F\right)w_{1,1}}\\ +\beta\dfrac{\left(1-\mu^F\right)w_{3,2}}{\mu^F+\left(1-\mu^F\right)w_{1,2}}\dfrac{\left(1-\mu^F\right)w_{2,1}}{\mu^F+\left(1-\mu^F\right)w_{1,1}}\end{array}\right), \tag{27}$$

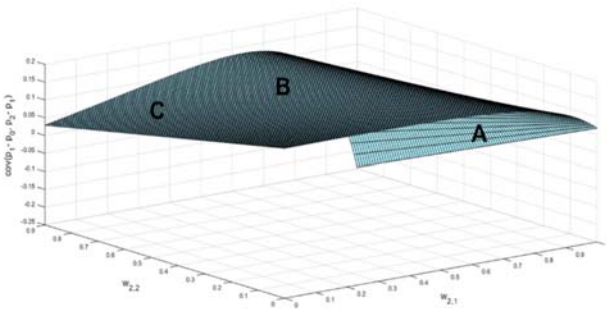
$$c=\frac{\left(1-\mu^F\right)w_{2,2}}{\mu^F+\left(1-\mu^F\right)w_{1,2}}-\frac{\left(1-\mu^F\right)w_{2,1}}{\mu^F+\left(1-\mu^F\right)w_{1,1}}+2\beta\frac{\left(1-\mu^F\right)w_{3,2}}{\mu^F+\left(1-\mu^F\right)w_{1,2}}\frac{\left(1-\mu^F\right)w_{2,1}}{\mu^F+\left(1-\mu^F\right)w_{1,1}}\quad, \tag{28}$$

$$d=\frac{\left(1-\mu^F\right)w_{2,1}}{\mu^F+\left(1-\mu^F\right)w_{1,1}}\left(\frac{\left(1-\mu^F\right)w_{2,2}}{\mu^F+\left(1-\mu^F\right)w_{1,2}}+\beta\frac{\left(1-\mu^F\right)w_{3,2}}{\mu^F+\left(1-\mu^F\right)w_{1,2}}\frac{\left(1-\mu^F\right)w_{2,1}}{\mu^F+\left(1-\mu^F\right)w_{1,1}}\right), \tag{29}$$

$$e=\frac{\left(1-\mu^F\right)w_{2,2}}{\mu^F+\left(1-\mu^F\right)w_{1,2}}+2\beta\frac{\left(1-\mu^F\right)w_{3,2}}{\mu^F+\left(1-\mu^F\right)w_{1,2}}\frac{\left(1-\mu^F\right)w_{2,1}}{\mu^F+\left(1-\mu^F\right)w_{1,1}}\,. \tag{30}$$

Eq. (24) shows that the short-term correlation of the stock returns increases with the weight of the sentiment signal ( $w_{2,2}$ ) and the extrapolative signal ( $w_{3,2}$ ) at date 2, but decreases with the weight of the sentiment signal at date1 ( $w_{2,1}$). Forthermore, we find that whether the short-term stock returns show a momentum effect or a reverse effect still depends on the change in the weight of the sentiment signal. To develop a better feel for the short-term correlation properties of stock returns, we give a numerical example in Figure 6. For simplicity, assume that the weight of sophisticated investors on value signal at date 2 is fixed and we set $w_{1,2}$=0.1, and the remaining parameters are chosen as in Figure 1. Part A of Figure 6 shows that when the weight of sophisticated investors on the sentiment signal at date 1 is sufficiently large, the reversal effect is dominant. Increasing the weight of sophisticated investors on the sentiment signal at date 2 can increase the reverse effect. Part B shows that when the weight of sophisticated investors on the sentiment signal at date 1 is at an intermediate level, the momentum effect is dominant. Increasing the weight of sophisticated investors on the sentiment signal at date 2 would increase the momentum effect. And Part C shows that when the weight of sophisticated investors on the sentiment signal at date 1 is sufficiently low, stock returns exhibit a momentum effect and increasing the weight of sophisticated investors on the sentiment signal at date 2 would decrease the momentum effect.

**Figure 6. The short-term covariance of stock returns caused by investor sentiment and extrapolation**




*Note: The figure illustrates the sensitivity of short-term covariance of stock returns ($Cov(p_1 - p_0, p_2 - p_1)$) to sophisticated investors' weight on sentiment signal at date 1 ($w_{2,1}$) and date 2 ($w_{2,2}$). The parameters are chosen as follows:*$\sigma_\varepsilon{}^2 = 0.2, \sigma_\eta{}^2 = 0.25, Q = 1, \gamma = 0.1, \mu^F = 0.3, \beta = 0.5, \rho_1 = 0.1$ *and* $w_{1,2}$*=0.1.*

## *3.3 The volatility of stock price changes*

In this section, we further investigate how investor sentiment and extrapolation influence the stock volatility and the combined effect of investor sentiment and extrapolation on stock volatility.

Let us first consider in our model the mechanism by which extrapolation affects the volatility of the stock price change. From Eq. (16), the volatility of stock price change at date 2 can be written as

$$Var\left(p_2^{**} - p_0^{**}\right) = \left(1 + f^{**2}\right)\sigma_\varepsilon^2 \tag{31}$$

where $f^{**} = 1 + \frac{\beta\left(1-\mu^F\right)w_{3,2}{}^{**}}{\mu^F + \left(1-\mu^F\right)w_{1,2}{}^{**}}$. We can find that extrapolation leads to excessive volatility of the stock price change (i.e., $Var\left(p_2^{**} - p_0^{**}\right) > 2\sigma_\varepsilon^2$), and the stronger the weight of sophisticated investors on extrapolative signal, the higher the volatility of the stock. This is due to the amplification term $\frac{\beta\left(1-\mu^F\right)w_{3,2}{}^{**}}{\mu^F + \left(1-\mu^F\right)w_{1,2}{}^{**}}$ resulting from the extrapolating endogenous stock returns (Atmaz,2022). Following the positive (negative) change in the stock price, the investor expects future stock returns to be higher (lower) and thus pushes the stock price further up (down), and this additional stock price fluctuation leads to higher stock return volatility.

Next, we examine how investor sentiment influences the stock volatility. From Eq. (15), we can write the volatility of the stock price change at date 2 as:

$$Var\left(p_2^* - p_0^*\right) = 2\sigma_\varepsilon^{\ 2} + 2h^{*2}\left(\frac{\sigma_\varepsilon^2}{\sigma_\eta^2}\right)\sigma_\varepsilon^2 + 2h^*\left(\rho_1 + \rho_2\right)\left(\frac{\sigma_\varepsilon}{\sigma_\eta}\right)\sigma_\varepsilon^2, \tag{32}$$

where $h^* = \dfrac{\left(1-\mu^F\right)w_{2,2}^{\ *}}{\mu^F + \left(1-\mu^F\right)w_{1,2}^{\ *}}$, $\rho_1$ and $\rho_2$ are the correlation coefficients between $\varepsilon_1$ and $\eta_1$, $\varepsilon_2$ and $\eta_2$, respectively. Eq. (32) shows that investor sentiment can also lead to excessive volatility of the stock price change (i.e., $Var\left(p_2^* - p_0^*\right) > 2\sigma_\varepsilon^{\ 2}$), and the stronger the weight of sophisticated investors on sentiment signal, the greater the excessive volatility of the stock price change. In our model, we see that its influence on stock volatility enters through two channels. The first has already been identified in extant research on investor sentiment and is due to the amplification term $2h^{*2}\left(\sigma_\varepsilon^2/\sigma_\eta^2\right)$ arising from the impact of investor sentiment on stock price. Following optimistic(pessimistic) investor sentiment, the investor expects future returns to be higher (lower) and thus pushes the stock price further up (down), and this additional stock price fluctuation leads to higher return variance. However, the second channel is novel to our model considering the cash-flow news shock on investor sentiment, and is due to the amplification term $2h^*\left(\rho_1+\rho_2\right)\left(\sigma_\varepsilon/\sigma_\eta\right)$ arising from sentiment update. This is because the greater the cash-flow news shock on investor sentiment, the more investor sentiment fluctuates and, in turn, the more the stock price fluctuates, leading to higher stock volatility.

We now explore the combined effect of the investor sentiment and the extrapolation on stock volatility. We can write the volatility of the stock price change at date 2 as

$$Var\left(p_2 - p_0\right) = \left(1+f^2\right)\sigma_\varepsilon^2 + \left(g^2 + h^2\right)\left(\frac{\sigma_\varepsilon^2}{\sigma_\eta^2}\right)\sigma_\varepsilon^2 + 2\left(fg\rho_1 + h\rho_2\right)\left(\frac{\sigma_\varepsilon}{\sigma_\eta}\right)\sigma_\varepsilon^2, \tag{33}$$

where

$$f = 1 + \beta\frac{\left(1-\mu^F\right)w_{3,2}}{\mu^F + \left(1-\mu^F\right)w_{1,2}}, \tag{34}$$

$$g = \frac{\left(1-\mu^F\right)w_{2,2}}{\mu^F + \left(1-\mu^F\right)w_{1,2}} + \beta\frac{\left(1-\mu^F\right)w_{3,2}}{\mu^F + \left(1-\mu^F\right)w_{1,2}}\frac{\left(1-\mu^F\right)w_{2,1}}{\mu^F + \left(1-\mu^F\right)w_{1,1}}, \tag{35}$$

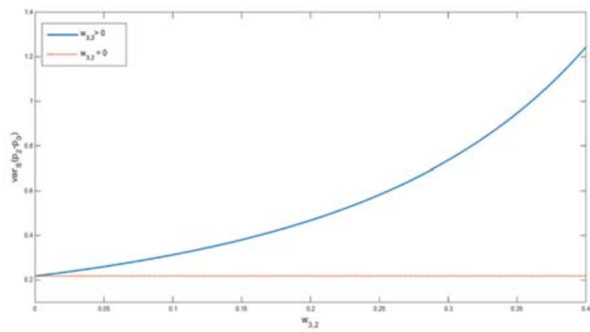
$$h = \frac{\left(1-\mu^F\right)w_{2,2}}{\mu^F + \left(1-\mu^F\right)w_{1,2}}. \tag{36}$$

From Eq. (33), we can find that both the weights of sentiment signal（$w_{2,2}$）and extrapolative signal ($w_{3,2}$) positively impact the stock volatility. It is worth noting that extrapolative behavior can amplify the excess stock volatility caused by investor sentiment due to extrapolating endogenous stock returns. For convenience, we define the volatility of stock price change generated by investor sentiment as $Var_S\left(p_2 - p_0\right)$, which can be given by

$$Var_S\left(p_2 - p_0\right) = \left(g^2 + h^2\right)\left(\frac{\sigma_\varepsilon^2}{\sigma_\eta^2}\right)\sigma_\varepsilon^2 + 2\left(fg\rho_1 + h\rho_2\right)\left(\frac{\sigma_\varepsilon}{\sigma_\eta}\right)\sigma_\varepsilon^2 . \tag{37}$$

Figure 7 illustrates the effect of extrapolation on stock volatility generated by investor sentiment, and $\rho_2 = \rho_1 = 0.1$, $w_{1,1} = 0.1$ and $w_{2,2} = 0.6$, the remaining parameters are chosen as those in Figure 1. Figure 7 shows that, given the weight of sophisticated investors on sentiment signal, stock volatility generated by investor sentiment increases monotonically in the weight of the extrapolative signal.

**Figure 7. Effects of extrapolation on the stock volatility generated by investor sentiment**



*Note: The figure plots the stock volatility driven by investor sentiment,* $Var_S(p_2 - p_0)$, *against the sophisticated investors' weight on extrapolative signal,* $w_{3,2}$.*The dotted line corresponds to the stock volatility driven by investor sentiment when sophisticated investors pay no attention to extrapolative signal. The parameters are chosen as follow:* $\sigma_\varepsilon{}^2 = 0.2, \sigma_\eta{}^2 = 0.25, Q = 1, \gamma = 0.1, \mu^F = 0.3, \beta = 0.5, \rho_2 = \rho_1 = 0.1, w_{1,1} = 0.1$ *and* $w_{2,2} = 0.6$.

# 4. Empirical Analysis

In the previous sections, we develop an equilibrium model featuring investor sentiment and extrapolation, and show that (1) stock price deviation from fundamental value increases in both investor sentiment and extrapolation, and the direction and the magnitude of stock price deviation depend on relative strength of sentiment signal and extrapolative signal; (2) the short-term correlation of stock returns increases in both the sentiment signal weight and extrapolative signal weight, but decreases in the lagged sentiment signal weight; (3) stock volatility increases in the weight of both the sentiment signal and the extrapolative signal, and extrapolative behavior exacerbates the sentiment-driven stock volatility. This section tests these three model predictions. Our sample contains all Chinese A-share firms listed on the Shanghai Stock Exchange and Shenzhen Stock Exchange with available stock return and company balance-sheet data from *CSMAR database*. The sample period spans from January 2000 to December 2021. We exclude the financial firms and special treatment (*ST*) firms in the sample.

### 4.1 Investor sentiment, extrapolation and stock price deviation

A prediction of the model is that both investor sentiment and extrapolation positively affect stock price deviation, thus, the magnitude and direction of stock price deviation is determined by the relative strength of the sentiment signal and the extrapolative signal. In this section, we test the prediction by portfolio sorts and Fama–MacBeth regressions.

We measure stock price deviation as the difference between the observed stock price and the predicted intrinsic value of individual stock. Following Rhodes-Kropf *et al*. (2005), we estimate the intrinsic value of a firm based on size, net income, and leverage and run the following regression:

$$LnQ_{i,t} = \alpha_0 + \alpha_1 LnSize_{i,t} + \alpha_2 AbsNI_{i,t} + \alpha_3 D_{(NI_{i,t}<0)} \times AbsNI_{i,t} + \alpha_4 Lev_{i,t} + \varepsilon_{i,t} \tag{38}$$

Where $LnQ_{i,t}$ is the logarithm market value of stock *i* in month *t*, $LnSize_{i,t}$ is the logarithm total assets of stock *i* at the end of the previous fiscal year, $NI_{i,t}$ is net income of stock *i* at the end of the previous fiscal year, $AbsNI_{i,t}$ is the logarithm of the absolute value of net income, $D_{(NI_{i,t}<0)}$ is a dummy variable that equals one when net income is negative and zero otherwise. $Lev_{i,t}$ is the leverage ratio of stock *i* at the end of the previous fiscal quarter. The fitted value ($LnQ_{i,t}^{f}$) from the regression represents the value of the stock justified by its current financial fundamentals, and the difference between $LnQ_{i,t}$ and $LnQ_{i,t}^{f}$ can be treated as the stock price deviation, denoted by $Dev_{i,t}$. $Dev_{i,t} > 0$ suggests that the stock price is overvalued, while $Dev_{i,t} < 0$ indicates that the stock price is undervalued.

According to Aboody *et al*. (2018), we use overnight returns to measure individual stock investor sentiment and compute the investor sentiment for stock *i* in month *t*, denoted by $Sent_{i,t}$, as the average daily investor sentiment during month *t*. More specifically, the daily investor sentiment index of stock *i* on day $d$ in month $t$, denoted by $Sent_{i,t,d}$, is calculated as follows:

$$Sent_{i,t,d} = \left(O_{i,t,d} - C_{i,t,d-1}\right) / C_{i,t,d-1} \quad , \tag{39}$$

Where $O_{i,t,d}$ is the opening price of stock *i* on day *d* in month *t*, and $C_{i,t,d-1}$ is the closing price of stock *i* on day *d-1* in month *t*. All opening and closing prices are adjusted for stock splits, stock dividends, and cash dividends.

We measure the extrapolation as the stock's cumulative return over months *t−12* to *t−2*, and the extrapolation index is as follows:

$$Extrap_{i,t} = \prod_{j=2}^{12}\left(1 + R_{i,t-j}\right) - 1, \tag{40}$$

Where $R_{i,t-j}$ represents the returns of stock $i$ in month *t-j*.

To remove the common influence between investor sentiment, extrapolation and market trend, we therefore perform the following orthogonalization of investor sentiment and extrapolation indexes:

$$Sent_{i,t} = \alpha_0 + \alpha_1 Rmrf_t + \alpha_2 Extrap_{i,t} + \varepsilon_{i,t} \,, \tag{41}$$

$$Extrap_{i,t} = \alpha_0 + \alpha_1 Rmrf_t + \alpha_2 Sent_{i,t} + \varepsilon_{i,t} \,, \tag{42}$$

where $Rmrf_t$ is the market excess return in month *t*. The residual in Eq. (41), labeled with $Sent^{\perp}{}_{i,t}$, can be the individual stock investor sentiment index. And the residual from Eq. (42), denoted by $Extrap^{\perp}{}_{i,t}$, is regarded as the extrapolation index.

Once we have constructed the indexes of investor sentiment, extrapolation and stock price deviation as described above, we examine the combined effect of investor sentiment and extrapolation on stock price deviation by portfolio sorts and Fama–MacBeth regressions.

Table C1 in the Online Appendix C reports the stock price deviation for portfolios double-sorted by investor sentiment and extrapolation. According to Table C1, we uncover that the stock price is overvalued among stocks with relatively more optimistic investor sentiment and higher extrapolative belief, but undervalued among stocks with relatively more pessimistic investor sentiment and lower extrapolative belief. Moreover, for the stocks with certain extrapolative belief, stock price deviation increases with investor sentiment, implying that optimistic (pessimistic) investor sentiment leads to stock price overvaluation (undervaluation). For stocks with a certain investor sentiment, stock price deviation increases with extrapolative belief, indicating that higher (lower) extrapolation will push stock prices (down).

Results from portfolio sorts provide preliminary supportive evidence for the model prediction that both investor sentiment and extrapolation have positive effects on stock price deviation. To identify the quantitative impacts of investor sentiment and extrapolation on the stock price deviation precisely, we further perform stock-level Fama–MacBeth regressions to complement our previous results. Table C2 in the Online Appendix C reports the results. We start in column (1) by looking at Fama-French three-factors, *Rmrf, Smb, Hml*. Compared with the column (1), it can be seen that the coefficient of the investor sentiment index ($Sent^{\perp}$) is 0.3037 (t=15.04) in column (2), which is significantly different from zero at the 1% level. The evidence suggests optimistic investor sentiment could increase (decrease) the degree of stock price overvaluation (undervaluation). In column (3), the explanatory variable are Fama-French three-factors and the extrapolation index ($Extrap^{\perp}$). It also can be seen that the coefficient of the extrapolation index ($Extrap^{\perp}$) is positive and the t-statistic is significant, suggesting that when the extrapolation index ($Extrap^{\perp}$) is high, the degree of stock price overvaluation (undervaluation) are high (low). However, the t-statistic of this variable is 2.85, and the $Avg.adj.R^2$ is also lower than for column (2), with a value of 0.2465. The results imply that investor sentiment has a more significant impact on the stock price deviation. Column (4) shows that adding the investor sentiment index ($Sent^{\perp}$) and the extrapolation index ($Extrap^{\perp}$) to Fama-French three-factors increases the explanatory of the regression. Moreover, both the investor sentiment index ($Sent^{\perp}$) and the extrapolation index ($Extrap^{\perp}$)

are significant with the t-statistics of 12.79 and 20.13, respectively. The results show that both the investor sentiment index ($Sent^{\perp}$) and the extrapolation index ($Extrap^{\perp}$) indeed have significant impacts on the stock price deviation, and the effects of the investor sentiment index ($Sent^{\perp}$) and the extrapolation index ($Extrap^{\perp}$) on stock price deviation are not subsumed by each other.

### 4.2 Sentiment beta, extrapolation beta and stock return correlation and stock volatility

Section 3 shows that both the weights of sophisticated investors on the sentiment signal and the extrapolative signal positively affect short-term stock return correlation and stock volatility. Therefore, given the fraction of sophisticated investors, both the stock loadings on investor sentiment and extrapolation have positive impact on short-term stock return correlation and stock volatility. To test this prediction of the model, we estimate the measure of investor sentiment sensitivity and extrapolation sensitivity as follows:

$$R_{i,t} - R_{f,t} = \beta_0 + \beta^s \Delta Sent^{\perp}{}_{i,t} + \beta^e \Delta Extrap^{\perp}{}_{i,t} + \varepsilon_{i,t}\,, \tag{42}$$

where $R_{i,t}$ is the return of stock *i* in month *t*, $R_{f,t}$ is the risk-free rate of return in month *t*, $\Delta Sent^{\perp}{}_{i,t}$ is the change of the investor sentiment of stock *i* in month *t*, and $\Delta Extrap^{\perp}{}_{i,t}$ is the change of the extrapolative belief of stock *i* in month *t*. Following the convention, we use standard rolling-window regression based on past-36-month observations and obtain the sentiment beta, denoted by $\beta^s$, and extrapolation beta, denoted by $\beta^e$, each month.

We further define the short-term correlation of stock returns as the correlation between stock monthly excess return and the past 1-month excess return of the stock. Specifically, we estimate the short-term return correlation of stock *i* in month *t*, denoted by $Corr_{i,t}$, using standard rolling-window correlations based on past-36-month observations. Furthermore, we calculate the idiosyncratic volatility of stock return stock *i* in month *t*, denoted by $IVOL_{i,t}$, as the standard deviation of daily excess returns relative to Fama-French three factors during month *t*.

Table C3 in the Online Appendix C presents summary statistics and the correlation coefficients across the measures of sentiment beta, extrapolation beta, short-term correlation of stock returns and idiosyncratic volatility. Panel A of Table C3 reports the time-series average of the cross-sectional mean, standard deviation, maximum, median and minimum statistics of the variables introduced above. The mean of sentiment beta is 0.1488 and its volatility is 0.1755. The mean of extrapolation beta is 0.1055 and its volatility is 0.1707. A casual observation shows that the short-term correlation of stock returns ($Corr$) varies from negative to positive values, indicating that short-term stock returns may exhibit a momentum effect or a reversal effect. Panel B reports the time-series average of the cross-sectional correlation estimates. Both sentiment beta index ($\beta^s$) and extrapolation beta index ($\beta^e$) are significantly correlated with short-term correlation of stock returns with the correlation coefficients of 0.0647 and 0.0244, respectively. Moreover, $\beta^s$ and $\beta^e$ are positively correlated with idiosyncratic volatility ($IVOL$). This confirms the positive effects of sentiment beta and extrapolation beta on the short-term stock return correlation and idiosyncratic volatility in the univariate test.

Next, we run Fama-MacBeth regressions of stock return correlation on sentiment beta and extrapolation beta. Table C4 in the Online Appendix C reports the results. We start in column (1) by looking at the lagged correlation of stock returns, denoted by $Corr_{i,t-1}$, and Fama-French three factors. Compared with the column (1), it can be seen that the coefficients of the sentiment beta and the lagged sentiment beta are 0.3149 (t=14.47) and -0.3106 (t=14.38), respectively, in column (2), which are both significantly different from zero at the 1% level. The evidence confirms the model's prediction that the short-term correlation of stock returns increases in the weight of sentiment signal, but decreases in the lagged weight of sentiment signal. In column (3), the explanatory variables are the lagged correlation of stock returns, Fama-French three factors and the extrapolation beta. It also can be seen that the extrapolation beta coefficient is positive and the t-statistic is significant, confirming the model's prediction that the weight of extrapolative signal is positively related to short-term correlation of stock returns. Column (4) shows that adding sentiment beta, lagged sentiment beta and extrapolation beta to the lagged correlation of stock returns, Fama-French three factors increases the explanatory of the regression. Moreover, both the coefficients of $\beta^s_{i,t}$ and $\beta^e_{i,t}$ remain significantly positive at the 1% level with a magnitude of 0.3081 (t=14.07) and 0.0079 (t=5.05), respectively. Meanwhile, the coefficient on $\beta^s_{i,t-1}$ is still significantly negative at the 1% level with a magnitude of -0.3097 (t=14.35). Thus, extrapolation (investor sentiment) still exerts additional effects on the short-term correlation of stock returns after controlling for sentiment (extrapolation) and other factors.

We further examine the effect of sentiment beta and extrapolation beta on stock return volatility, and Table C5 in the Online Appendix C reports the results. We start in column (1) by looking at the lagged volatility of stock returns and Fama-French three factors. Compared with the column (1), it can be seen that the coefficient of sentiment beta is 0.0012 (t=7.46) in column (2), which is significantly different from zero at the 1% level. The evidence supports the model's prediction that the weight of sentiment signal positively influences the volatility of stock returns. In column (3), the explanatory variables are the lagged volatility of stock returns, Fama-French three factors and the extrapolation beta. It also can be seen that the extrapolation beta coefficient is positive, and t-statistic is significant, confirming the model's prediction that the weight of extrapolative signal relates positively to volatility of stock returns. Column (4) shows that adding the investor sentiment beta and the extrapolation beta to the lagged volatility of stock returns and Fama–French three factors increases the explanatory of the regression. Moreover, both the coefficients on sentiment beta and extrapolation beta remain significantly positive at the 1% level with a magnitude of 0.0005 (t=1.93) and 0.0079 (t=0.0009), respectively. Thus, extrapolation (investor sentiment) still exerts additional effects on the volatility of stock returns after controlling for the sentiment (extrapolation) and other factors. The above evidence is consistent with the model prediction the both the weights of sentiment signal and extrapolative signal positively affect the stock return volatility.

Finally, we examine the effect of extrapolative behavior on the stock return volatility driven by investor sentiment. Specifically, we calculate the stock return volatility driven by investor sentiment of stock *i* in month *t*, denoted by $VOL^S_{i,t}$, as the standard deviation of daily stock returns generated by daily investor sentiment during month t. Table C6 in the Online

Appendix C reports the results. We start in column (1) by looking at the lagged volatility of stock returns driven by investor sentiment, denoted by $VOL^{S}_{i,t-1}$, and Fama-French three factors. Compared with the column (1), it can be seen that the sentiment beta coefficient is 0.0010 (t=6.456) in column (2), which is significantly different from zero at the 1% level. In column (3), the explanatory variables are the lagged volatility of stock returns driven by investor sentiment, Fama-French three factors and the extrapolation beta. It can be seen that the extrapolation beta coefficient is positive and t-statistic is significant, confirming the model's prediction that the extrapolative signal weight positively impact on the stock return volatility driven by investor sentiment. Column (4) shows that adding sentiment beta and extrapolation beta to the lagged volatility of stock returns driven by investor sentiment, Fama-French three factors increase the explanatory of the regression. Moreover, the extrapolation beta coefficient remains significantly positive at the 1% level with a magnitude of with 0.0006 (t=4.23). Thus, the above evidence is consistent with the model's prediction that extrapolative behavior exacerbates sentiment-driven stock volatility.

## 5. Conclusions

In this paper, we investigate the combined effects of investor sentiment and extrapolation on asset pricing by assuming that sophisticated investors form their stock demand based on three signals: sentiment signal, extrapolative signal, and value signal. In our analysis, we demonstrate that the equilibrium quantities are driven by the relative strength of the sentiment signal and the extrapolative signal. We find that both investor sentiment and extrapolation lead to stock price deviation from fundamental value, and the direction and the magnitude of stock price deviation depend on the relative strength of the sentiment signal and the extrapolative signal. In addition, our model also predicts that both the weights of sentiment signal and extrapolation signal have a positive effect on the short-term correlation of the stock returns, while the lagged weight of sentiment signal negatively impact the correlation of stock returns. Furthermore, the model shows that both sentiment signal weight and extrapolative signal weight are positively related to volatility of stock price change, and more importantly, extrapolative behavior can amplify the stock return volatility driven by investor sentiment due to extrapolating endogenous stock returns.

We find evidence consistent with model's predictions: first, we find that the stock price is overvalued among stocks with optimistic investor sentiment and higher extrapolation belief but undervalued among stocks with pessimistic investor sentiment, and lower extrapolation belief, and both investor sentiment and extrapolation positively affect stock price deviation. Second, we show that the short-term correlation of stock returns increases with both sentiment beta and extrapolation beta, but decreases with lagged sentiment beta. Finally, we find that stock return volatility increases with both sentiment beta and extrapolation beta, and extrapolation beta positively influences the sentiment-driven stock return volatility.

Our findings may raise some interesting issues for further research. For example, we can assume that there is another type of investors who chase investor sentiment and build a multiple periods or even continuous model to analyze the combined effect of investor sentiment and extrapolation on asset pricing.

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